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IMPLEMENTING THE CROSS AMBIGUITY FUNCTION AND GENERATING GEOMETRY-SPECIFIC SIGNALS

by

Joe J. Johnson

September 2001

Thesis Advisor:	Herschel H. Loomis, Jr.
Second Reader:	Ralph D. Hippenstiel

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The first purpose of this thesis is to implement an efficient Cross Ambiguity Function (CAF) algorithm to compute the Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) between two sampled signals. Two CAF-related MATLAB functions were written and analyzed. One implements a "coarse" mode and a "fine" mode to accurately compute the TDOA and FDOA. The second plots different views of the resulting three-dimensional CAF surface.

The second purpose is to develop a program to generate geometry-specific signals. Some software packages can artificially embed constant TDOAs and FDOAs between two signals. In real-world emitter-collector geometries (one emitter and two separate collectors), however, movement of the emitter and/or collectors causes time-varying TDOAs and FDOAs. A MATLAB function was written to generate pairs of Binary-Phase-Shift-Keying signals according to user-defined signal parameters and Cartesian geometries. The resulting signal pairs have realistic TDOAs and FDOAs that vary with time according to geometry and relative motion.

Several signal pairs with different geometries are generated and input into the CAF functions, and the results are compared with theoretical TDOA and FDOA calculations. Finally, signals with low signal-to-noise ratios are generated to evaluate the CAF's ability to find Low Probability of Detection signals.

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IMPLEMENTING THE CROSS AMBIGUITY FUNCTION AND GENERATING GEOMETRY-SPECIFIC SIGNALS

Joe J. Johnson
Lieutenant Commander, United States Navy
B.S., United States Naval Academy, 1992

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September 2001

Author: Joe J. Johnson
Joe J. Johnson

Approved by: [illegible]
Herschel H. Loomis, Jr., Thesis Advisor

Ralph D. Hippenstiel
Ralph D. Hippenstiel, Second Reader

[illegible]
Jeffrey B. Knorr, Chairman
Department of Electrical and Computer Engineering

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EXECUTIVE SUMMARY

The location of radio frequency transmitters is critical to numerous applications. Many geolocation methods utilize the Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) between two receivers collecting the same transmission. One method of computing the TDOA and FDOA jointly is the Cross Ambiguity Function (CAF). In the discrete, sampled-time case, it is defined mathematically as:

$$CAF(\tau,k)=\sum_{n=0}^{N-1} s_1(n)s_2^*(n+\tau)e^{-j2\pi\frac{kn}{N}} \tag{1}$$

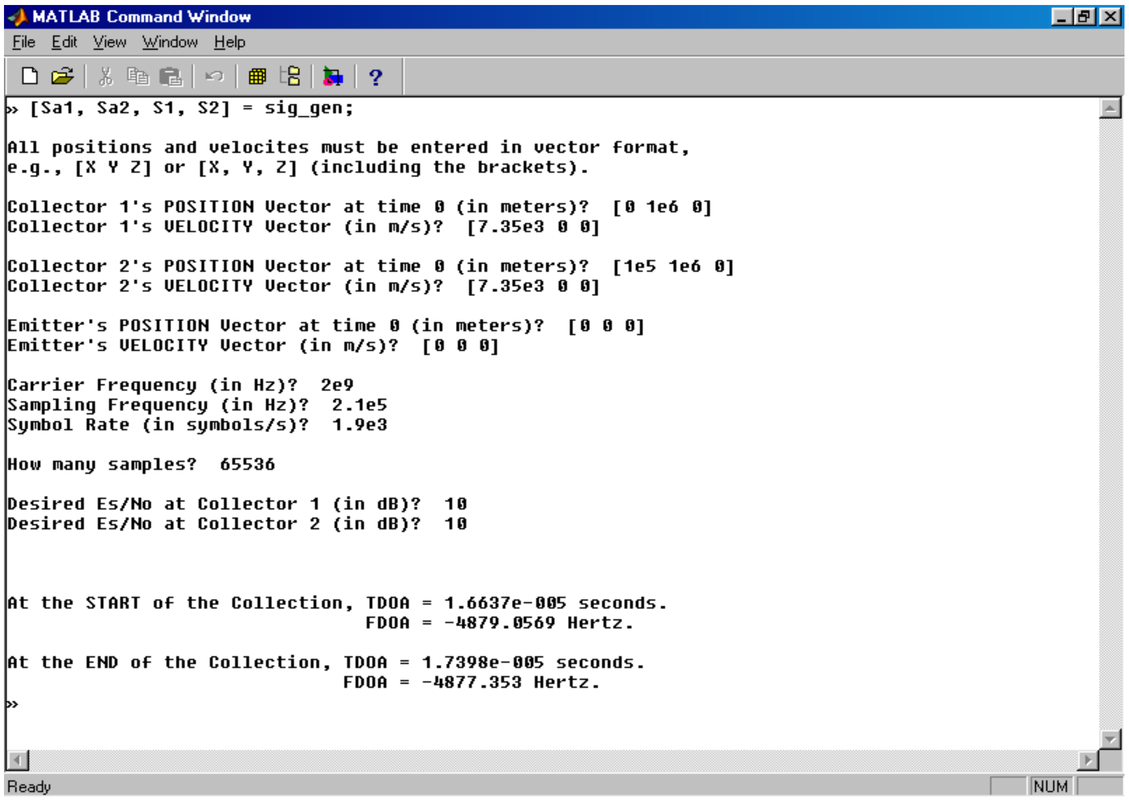
where s_1 and s_2 are sampled signals in *analytic signal* format, *N* is the total number of samples in s_1 and s_2, τ is time delay in *samples*, and $\frac{k}{N}$ is the frequency difference in *digital frequency*, or fraction of the sampling frequency. The magnitude of the CAF, or $|CAF(\tau,k)|$, will peak when τ and $\frac{k}{N}$ are equal to the embedded TDOA and FDOA, respectively, between the two signals s_1 and s_2. The first goal of this thesis was to implement Equation (1) to efficiently compute the TDOA and FDOA between two sampled signals.

There are three main ways to implement Equation (1) directly. The summation can be explicitly computed, or the terms can be rearranged in two different ways to force Equation (1) into the form of either a Discrete Fourier Transform or a cross-correlation. The three methods are evaluated and their computational complexities (in terms of floating point operations) are compared. The result is that even for a relatively small number of possible TDOAs and FDOAs, all three methods of direct computation are too costly. A better approach is to split the computations into two modes: "coarse" and "fine." The coarse mode produces a rough estimation of TDOA and FDOA by dividing the signals into smaller blocks for processing, which reduces the overall processing burden. The coarse estimates are then sent to the fine mode for refined computation.

Because the fine mode computes the CAF for a small number of possible TDOAs and FDOAs (i.e., for a few values surrounding the coarse estimates), the CAF can be implemented directly. From the analysis of the three methods described above, the explicit summation method is the most efficient. This approach was used to develop a MATLAB function, CAF.m, that takes two signal vectors and computes the associated TDOA and FDOA. Another program, CAF_peak.m, displays the resulting CAF surface in both 3-D and 2-D. Several pairs of signals with constant TDOAs and FDOAs were input into the programs to ensure that they worked.

The second goal of the thesis was to develop a MATLAB program that generates realistic signal sets. Some commercial software packages have the ability to embed only *constant* TDOAs and FDOAs between two signals. In real-world systems, however, the relative motion between emitters and collectors causes time-varying TDOAs and FDOAs. The program sig_gen.m was developed so that a user can define signal parameters (carrier frequency, sampling frequency, data rate, etc.) and a specific emitter-collector geometry in Cartesian, three-dimensional, coordinates. The generated signal sets represent realistic signals that have been transmitted from a system with the characteristics defined by the user. In this manner, one can use sig_gen.m to simulate real-world systems.

As an example, consider a ground-based transmitter with a pair of satellite collectors in a Low Earth Orbit (LEO) of 1000 kilometers. Figure (1) shows the MATLAB command window after sig_gen.m generates a pair of signals with this geometry. The theoretical values for TDOA and FDOA are shown at the bottom of the figure. Figure (2) shows the MATLAB command window after running CAF.m on the generated signals. Notice that the computed values of TDOA and FDOA, shown in Figure (2), compare favorably with the theoretical predictions in Figure (1). Finally, Figure (3) shows the 3-D plot of the resulting CAF surface and Figure (4) shows 2-D views that result from slices through the surface along the TDOA and FDOA axes. It is important to note that the CAF surface is for display only, since its peak occurs at un-interpolated values of TDOA and FDOA. This reduces the processing burden of creating



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» [Sa1, Sa2, S1, S2] = sig_gen;

All positions and velocites must be entered in vector format,
e.g., [X Y Z] or [X, Y, Z] (including the brackets).

Collector 1's POSITION Vector at time 0 (in meters)?  [0 1e6 0]
Collector 1's VELOCITY Vector (in m/s)?  [7.35e3 0 0]

Collector 2's POSITION Vector at time 0 (in meters)?  [1e5 1e6 0]
Collector 2's VELOCITY Vector (in m/s)?  [7.35e3 0 0]

Emitter's POSITION Vector at time 0 (in meters)?  [0 0 0]
Emitter's VELOCITY Vector (in m/s)?  [0 0 0]

Carrier Frequency (in Hz)?  2e9
Sampling Frequency (in Hz)?  2.1e5
Symbol Rate (in symbols/s)?  1.9e3

How many samples?  65536

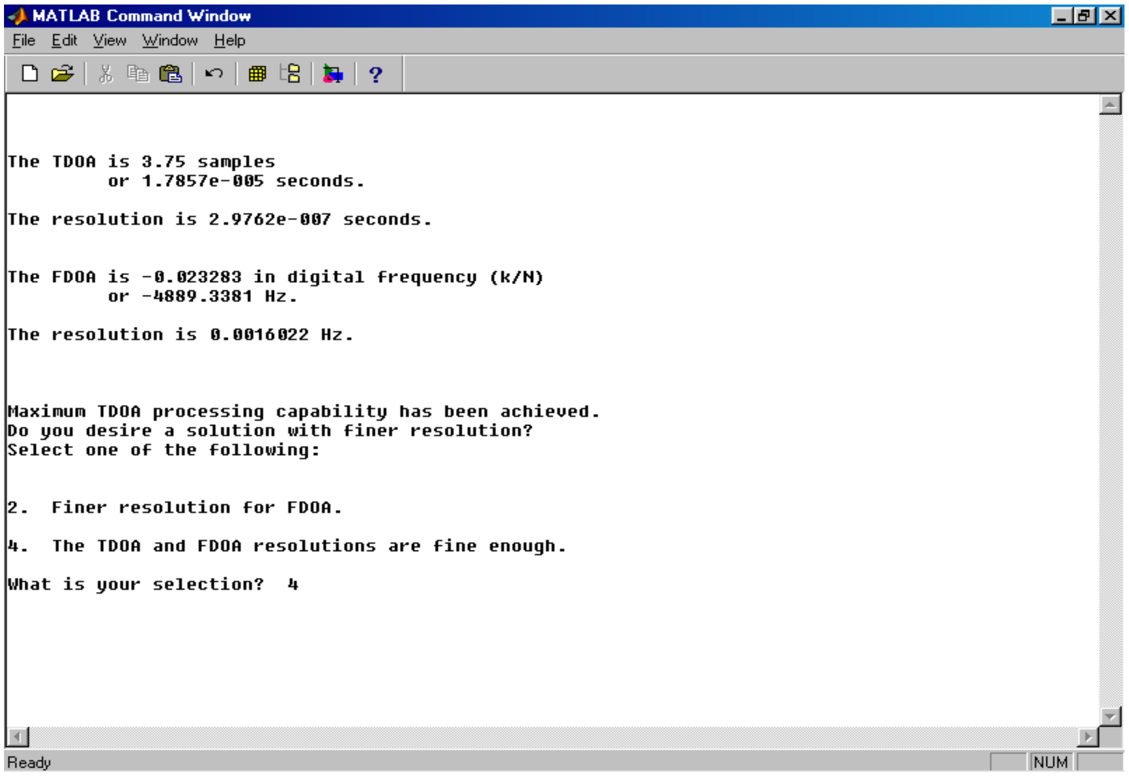
Desired Es/No at Collector 1 (in dB)?  10
Desired Es/No at Collector 2 (in dB)?  10



At the START of the Collection, TDOA = 1.6637e-005 seconds.
                                FDOA = -4879.0569 Hertz.

At the END of the Collection, TDOA = 1.7398e-005 seconds.
                              FDOA = -4877.353 Hertz.
»
```

Figure 1. Example Signal Set (LEO Satellite Collectors & Ground-Based Emitter).



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The TDOA is 3.75 samples
         or 1.7857e-005 seconds.

The resolution is 2.9762e-007 seconds.


The FDOA is -0.023283 in digital frequency (k/N)
         or -4889.3381 Hz.

The resolution is 0.0016022 Hz.



Maximum TDOA processing capability has been achieved.
Do you desire a solution with finer resolution?
Select one of the following:


2.  Finer resolution for FDOA.

4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4
```

Figure 2. CAF.m Results (LEO Satellite Collectors & Ground-Based Emitter).

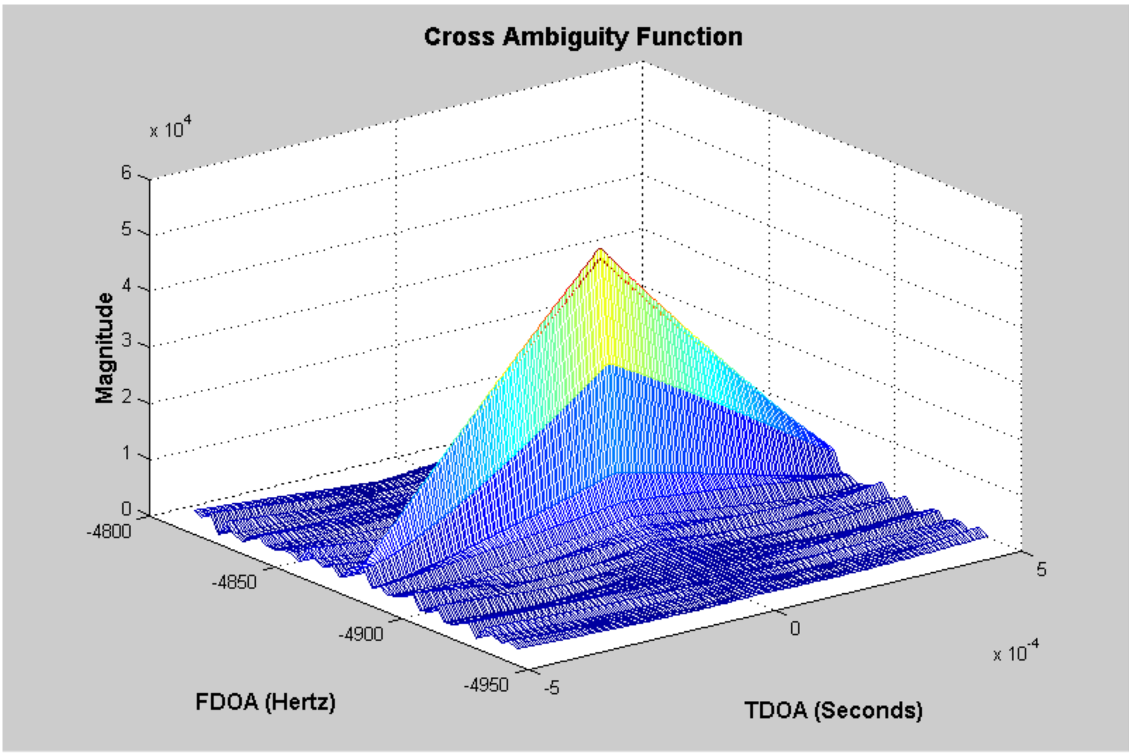

Figure 3. 3-D CAF Surface (LEO Satellite Collectors & Ground-Based Emitter).

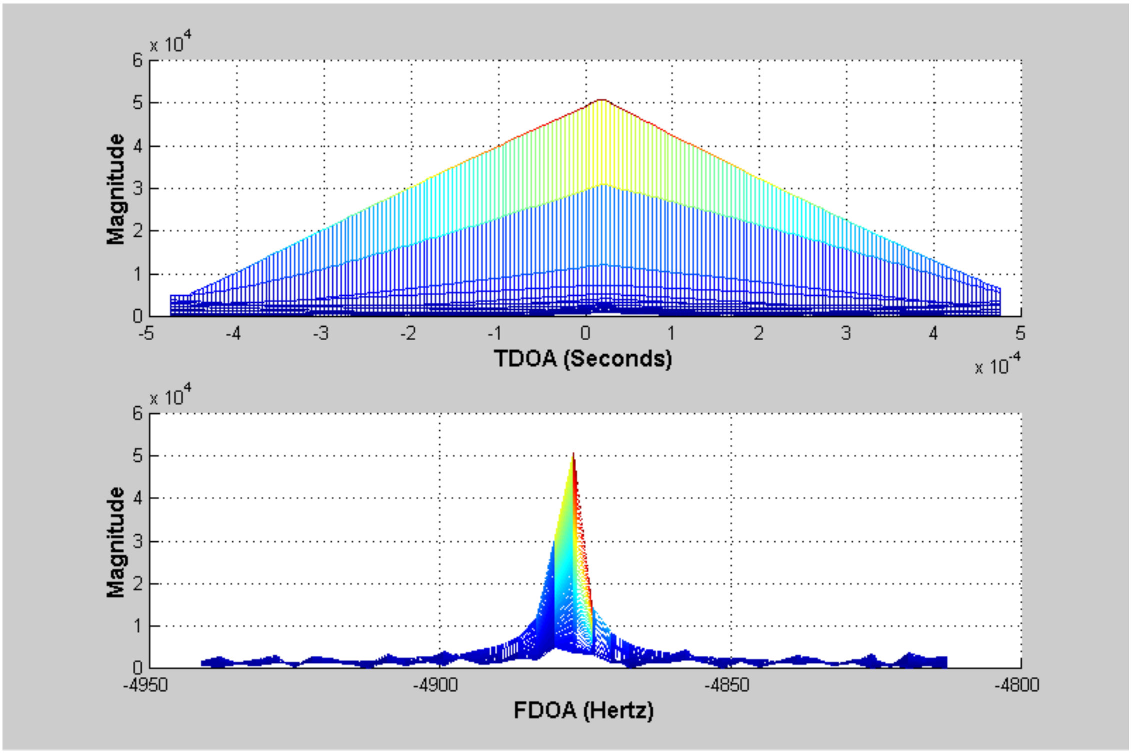

Figure 4. 2-D Cuts Through CAF Surface (LEO Satellite Collectors & Ground Emitter).

the surface. As a result, the TDOA & FDOA shown in Figures (3) and (4) do not correspond exactly to the more precise values computed by CAF.m and displayed in Figure (2).

A variety of other signal sets were also generated to ensure that the programs developed for this thesis operated correctly. The end result is that the goals of the thesis were clearly met. The CAF and signal generation software developed for this thesis provide a new capability for users to simulate real-world systems, generate realistic BPSK signals, and efficiently compute TDOAs and FDOAs – all on a standard desktop PC.

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I. INTRODUCTION

A. BACKGROUND

Accurate geolocation of radio frequency transmitters is critical to many applications, including Global Positioning and pinpointing the locations of hostile radar systems. Many geolocation methods utilize the Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) between two receivers collecting the same transmission. If there is no FDOA between two receivers (i.e., the difference in the two Dopplers is zero), then simple cross-correlation computations can uncover the resulting TDOA. However, in the more likely cases where relative motion exists between collectors and transmitters, the non-zero FDOAs preclude use of cross-correlation techniques. In these cases, TDOA and FDOA measurements must be calculated *jointly*. One way to accomplish this is to utilize the Cross Ambiguity Function (CAF).

There exist many signal generation software packages that can produce myriad signal types (Phase Shift Keying, Amplitude Shift Keying, etc.) with user-defined parameters such as carrier frequency, sampling frequency, symbol rate, and signal-to-noise ratio. Some of these packages can also embed time delays and frequency offsets between two signals. The limitation in these programs is that the embedded TDOAs and FDOAs are *constant*. This is not helpful in modeling real-world situations where relative motion between emitters and collectors causes a continuous change in geometry, and therefore leads to TDOAs and FDOAs that are *time-varying*.

B. OBJECTIVES

The main objective of this thesis was to develop the MATLAB code in Appendix A, which takes two sampled signals (i.e., transmissions from a single emitter received by two separate collectors) and estimates the associated TDOA and FDOA using CAF computations. In addition to TDOA and FDOA estimation, the CAF can also be used to *detect* signals. This feature is useful in evaluating the effectiveness of so-called Low Probability of Dectection (LPD) signals. The CAF is used in many real-world systems that have vast computer resources with which to do the computations. This software, however, brings the ability to perform CAF computations to the standard desktop PC.

The secondary focus of this thesis was to develop the MATLAB code in Appendix B, which generates pairs of sampled signals based upon signal parameters and emitter-collector geometries defined by the user. This will allow users to model any real-world system by creating signal sets that could be transmitted and collected by emitters and collectors in an associated geometry.

C. RELATED WORK

There exist numerous technical papers and articles on the CAF, and on algorithms that can be used to compute it. The vast majority of these papers refer back to [1], which is generally regarded as the seminal work on CAF processing techniques. Stein's paper, along with many others, describes an algorithm for efficient computation of the CAF. A significant search for references that deal specifically with *implementing* CAF algorithms turned up nothing. Searching the worldwide web for CAF programs uncovered a short MATLAB function that is part of a collection of free programs called the Time Frequency Toolbox for MATLAB [2]. Analysis of that CAF program, however, showed that it was rudimentary and incapable of processing signals that had greater than about 256 data elements. The CAF programs created as part of this thesis (Appendix A) are capable of handling signals with as many as 524,288 elements. The main program computes the CAF using the coarse mode algorithm described in [1], and a fine mode algorithm that was selected based upon an analysis of computational complexity.

As far as geometry-specific signal generation is concerned, there appeared to be no body of knowledge from which to draw upon. As mentioned in the previous section, there are some commercial software packages, including one from Statistical Signal Processing, Inc., that can embed constant TDOAs and FDOAs between two signals. The signal generation software developed for this thesis (Appendix B), however, allows a user to generate signals whose TDOAs and FDOAs, be they constant or time-varying, are consistent with the defined parameters and emitter-collector geometries. The algorithm used to generate these signals was developed and implemented through extensive trial and error by the author and his thesis advisor.

D. THESIS ORGANIZATION

The chapters of this thesis devoted to the CAF are organized as follows: Chapter II provides background information about the CAF, including basic definitions and requirements of the CAF's input signals. Chapter III evaluates the computational complexity (or cost) of three different ways in which the basic CAF can be directly implemented. Also, the code in Appendix A is thoroughly analyzed to describe the specific approach taken to implement the CAF. Finally, graphical and numerical results are displayed and discussed for several example signal sets that were input into the Appendix A programs.

There are two chapters devoted to geometry-specific signal generation. Chapter IV provides background information about Binary-Phase-Shift-Keying (BPSK) signals and the type of emitter-collector geometry that is modeled by the code. Also, equations that can be used to manually calculate TDOAs and FDOAs for known emitter-collector geometries are presented. Chapter V describes in detail the code in Appendix B, analyzing the technique used to create the signal sets. Also, several example sets of signals are generated, with their theoretical TDOAs and FDOAs calculated and compared to the actual values computed by the CAF code in Appendix A. Various cases are explored, including geometries that give different combinations of constant and time-varying TDOAs and FDOAs. Also, signal sets from some realistic geometries (e.g., satellite and airborne collectors) are analyzed and displayed. Finally, the detectability of LPD emitters is explored by showing how CAF results are affected by increasing the noise level. Chapter VI summarizes the findings of this thesis, and also discusses a number of extensions to this research that could be taken on by future students.

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II. THE CROSS AMBIGUITY FUNCTION

A. DEFINITION

In [1], the Cross Ambiguity Function (CAF) is mathematically defined as:

$$CAF(\tau, f) = \int_0^T s_1(t) s_2^*(t+\tau) e^{-j2\pi ft} dt, \qquad (2\text{-}1)$$

where s_1 and s_2 are continuous-time signals in *analytic signal* format (as defined in section B below), *T* is the integration period in seconds, *τ* is time delay in seconds, and *f* is the frequency offset in Hertz.

In order to shift Equation (2-1) into the discrete (or sampled) time domain, let $t = nT_s$ and $f = \frac{kf_s}{N}$, where T_s is the sample period, $f_s = \frac{1}{T_s}$ is the sampling frequency, *n* represents individual sample numbers, and *N* is the total number of samples. Inserting these values back into Equation (2-1) and simplifying yields the discrete form of the CAF:

$$CAF(\tau, k) = \sum_{n=0}^{N-1} s_1(n) s_2^*(n+\tau) e^{-j2\pi \frac{kn}{N}}, \qquad (2\text{-}2)$$

where s_1 and s_2 are sampled signals in *analytic signal* form, *N* is the total number of samples in s_1 and s_2, *τ* is time delay in *samples*, and $\frac{k}{N}$ is the frequency difference in *digital frequency*, or fraction of the sampling frequency. The magnitude of the CAF, or $|CAF(\tau, k)|$, will peak when *τ* and $\frac{k}{N}$ are equal to the embedded TDOA and FDOA, respectively, between the two signals s_1 and s_2. Note the assumption that the signal's presence has been previously detected, and subsequently collected as s_1 and s_2. The CAF itself is also capable of signal detection. This is discussed further in section V.C.

The code in Appendix A was developed to efficiently implement Equation (2-2). As will be shown in the next chapter, there are several different ways to implement Equation (2-2). Efficiency becomes a large factor because of the potentially huge range of TDOAs and FDOAs that must be searched. Equation (2-2) can uncover TDOAs in the range –N to N and FDOAs for *k* in the range $-\frac{N}{2}+1$ to $\frac{N}{2}$. To search the entire range of possible TDOAs and FDOAs would require $2N^2$ calculations of the CAF, an ominous task for large *N*!

B. ANALYTIC SIGNAL VS. COMPLEX ENVELOPE

In [1], Stein presents Equation (2-1) and then notes that the input signals s_1 and s_2 must be in *complex envelope* format. In actuality, the signals must be in *analytic signal* format. This is clearly an issue of semantics, but since a significant amount of time was lost attempting to compute the CAF on signals in *complex envelope* format, it is worthwhile to present the difference between *complex envelope* and *analytic signal*.

Bandpass signals have two-sided frequency spectra. Figure (2-1) depicts the spectral density (obtained by using the periodogram) of a Binary-Phase-Shift-Keying (BPSK) signal with carrier frequency f_0 = 1 MHz and sampled at f_s = 4 MHz. The periodogram is symmetric about the center of the frequency axis, with identical lobes appearing in the positive and negative frequency planes. When processing and analyzing a signal, it is generally common practice to deal only with the positive frequencies. Altering a signal such that only the positive side of the periodogram remains produces the *analytic signal*.

If *X(f)* is the spectrum of a continuous time bandpass signal, then the *analytic signal* is computed as:

$$X_a(f) = X(f) + j\hat{X}(f), \tag{2-3}$$

where $\hat{X}(f)$ is the Hilbert Transform of *X(f)*:

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f), \tag{2-4}$$

with the signum function defined as:

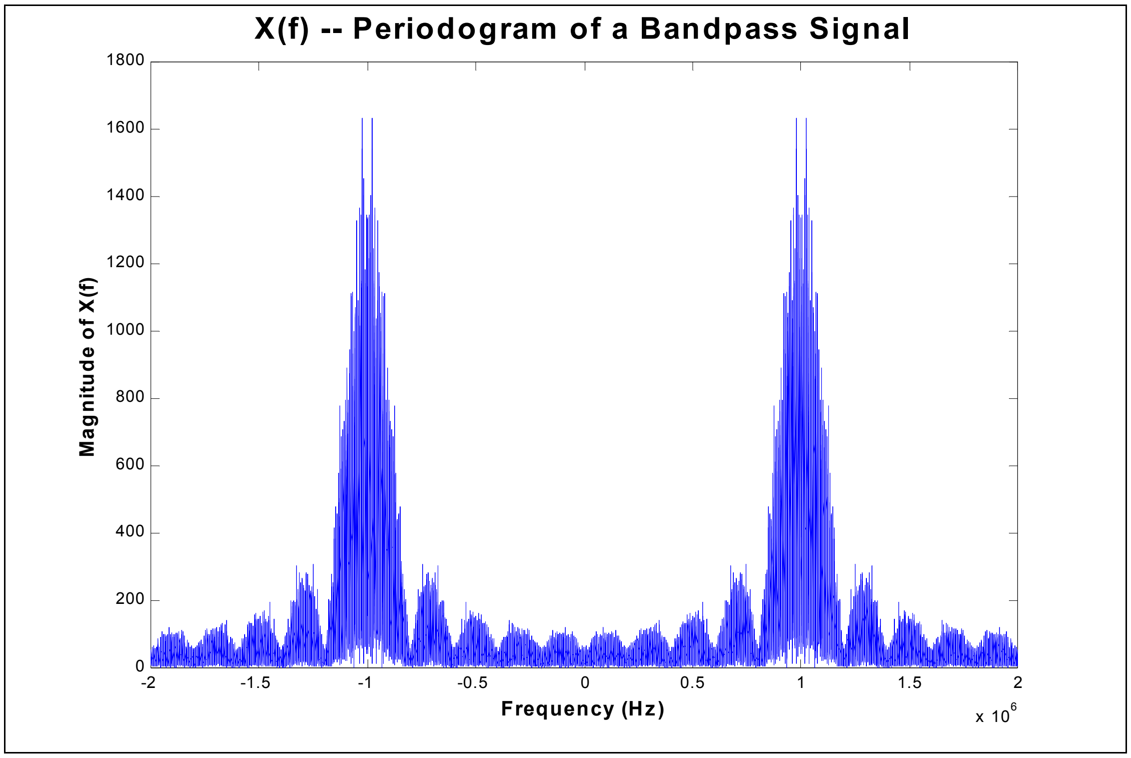

Figure 2-1. Periodogram of a Sampled Bandpass Signal.

$$\operatorname{sgn}(f) = \begin{cases} +1, & f > 0 \\ -1, & f < 0 \end{cases} \tag{2-5}$$

Substituting Equation (2-4) into Equation (2-3) and simplifying leads to [3]:

$$X_a(f) = \begin{cases} 2X(f), & f > 0 \\ 0, & f < 0 \end{cases} \tag{2-6}$$

Figure (2-2) shows the spectral density (periodogram) of the *analytic signal* of the sampled bandpass signal shown in Figure (2-1). Note that the *analytic signal* is indeed one-sided, and the magnitude is exactly twice that of the lobes in Figure (2-1).

Some applications require real bandpass signals to be represented as complex baseband signals. Known as the *complex envelope* of the signal, it simply entails shifting the *analytic signal* in the frequency domain by an amount equal to the carrier frequency, such that the single-sided spectrum is symmetric about $f = 0$. Using Fourier Transform properties, a shift in the frequency domain is accomplished by multiplication with a complex exponential. The complex envelope of a signal can therefore be represented as:

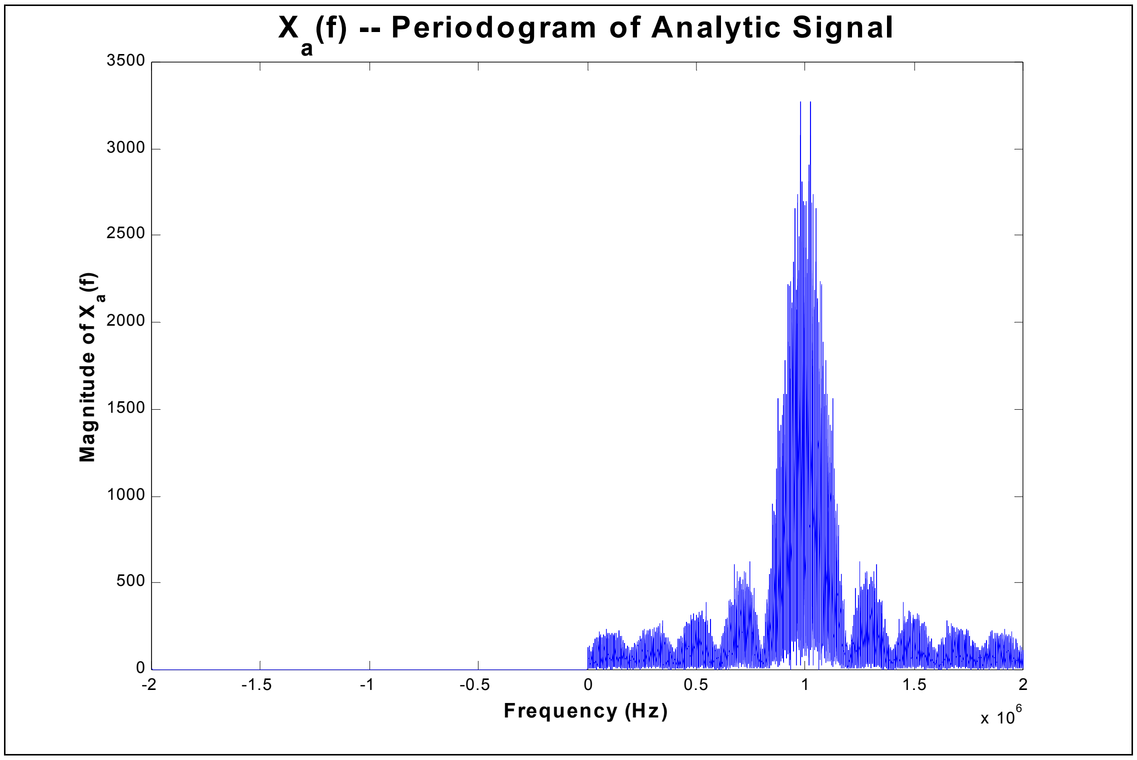

Figure 2-2. Periodogram of Analytic Signal.

$$\tilde{X}(f) = X_a(f)e^{-j2\pi f_0 t} \tag{2-7}$$

Continuing with the same example signal, Figure (2-3) shows the spectral density representation of the *complex envelope*. Notice that the resulting periodogram is indeed a replica of the *analytic signal*, but shifted down to baseband.

In order to work properly, the CAF requires input signals to be *analytic signal* representations. When *complex envelope* signals were used instead, extensive (and frustrating) experience showed that the CAF accurately computed TDOAs, but in every case the calculated FDOA was exactly zero. In retrospect, and in light of Figures (2-2) and (2-3), this result seems obvious. After all, when represented in *complex envelope* form, a signal's Doppler shift is effectively wiped out when the periodogram is shifted to baseband. This is not good since it is the difference of the Doppler shifts present in two signals that provides the FDOA! Therefore, computing the CAF on two *complex envelope* signals should always produce an FDOA equal to zero!

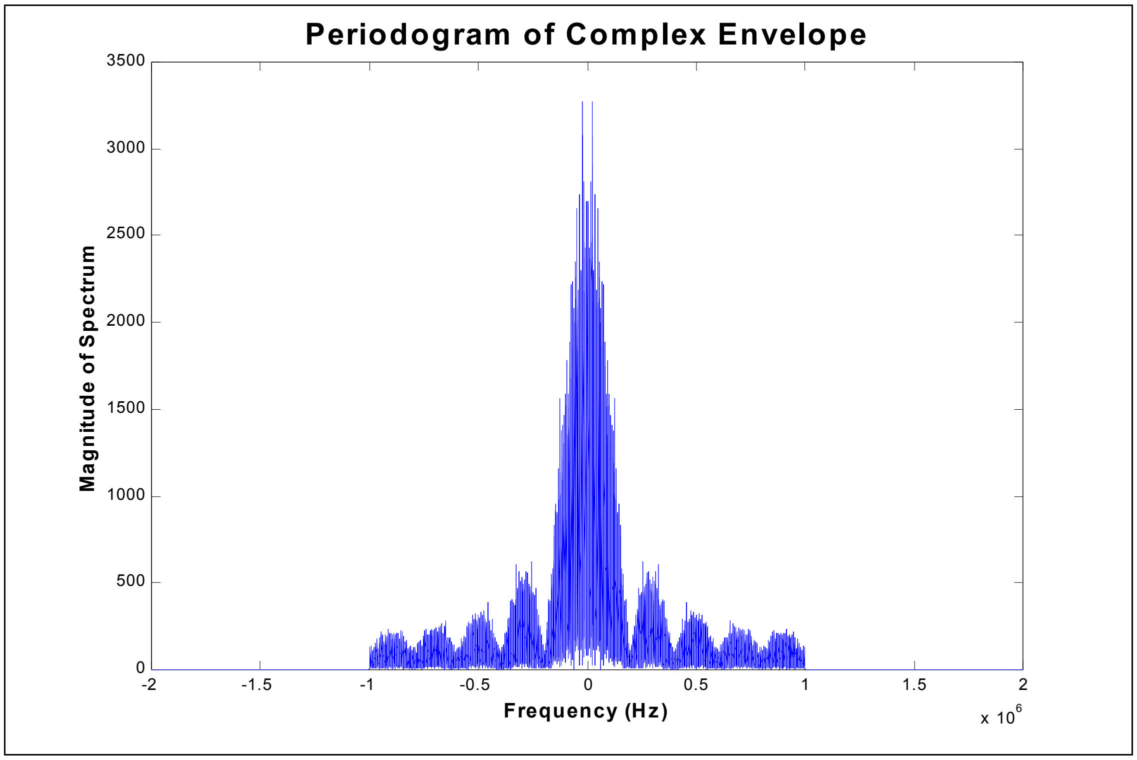

Figure 2-3. Periodogram of Complex Envelope.

Now that the CAF has been mathematically defined, Chapter III will analyze the computational complexity of three different methods that can be used to implement Equation (2-2) directly. Chapter III will also describe in detail the actual approach used by the MATLAB code in Appendix A to compute the CAF. Finally, Chapter III will summarize the results of running the code on some example signal sets.

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III. IMPLEMENTING THE CROSS AMBIGUITY FUNCTION

A. COMPUTATIONAL COMPLEXITY (COST) ANALYSIS

There are three main approaches to implementing the discrete CAF (Equation (2-2)) directly: utilizing the Discrete Fourier Transform, using the cross-correlation technique, and directly computing the summation. Each of these three methods has advantages and disadvantages, which will be discussed in the following subsections. Additionally, the complexity of the methods will be analyzed in terms of their "cost," or the total number of real multiply and real add operations required for each.

1. The Fast Fourier Transform Method

Looking closely at Equation (2-2), it clearly resembles the definition of the Discrete Fourier Transform (DFT) [4]:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{kn}{N}}, \qquad (3\text{-}1)$$

where

$$X(k) \equiv DFT\left[x(n)\right] \qquad (3\text{-}2)$$

The summation and the complex exponential term are the same for both Equations (3-1) and (2-2). By grouping the s_1 and s_2 terms in Equation (2-2),

$$CAF(\tau,k) = \sum_{n=0}^{N-1} [s_1(n) s_2^*(n+\tau)] e^{-j2\pi\frac{kn}{N}}, \qquad (3\text{-}3)$$

and realizing that $[s_1(n) s_2^*(n+\tau)]$ is analogous to *x(n)* in Equations (3-1) and (3-2), the CAF can be expressed as:

$$CAF(\tau,k) = DFT[s_1(n) s_2^*(n+\tau)] \qquad (3\text{-}4)$$

Using Equation (3-4) to calculate the CAF for all values of *τ* and *k*, an individual DFT computation is required for each value of *τ*. The DFT summation is normally computed using the Fast Fourier Transform (FFT) algorithm. The power of the FFT is that, for a given value of τ, it efficiently calculates the values associated with *every* value of k (i.e., all digital frequencies). Since k can take on values from $-\frac{N}{2}+1$ to $\frac{N}{2}$, the complete range of digital frequencies $\left(\frac{k}{N}\right)$ over which the FFT is calculated is approximately $-\frac{1}{2}$ to $\frac{1}{2}$. The main disadvantage with the FFT method is that for the vast majority of emitter-collector geometry and signal parameter combinations, the possible range of FDOAs is a very small subset of the full range of $-\frac{1}{2}$ to $\frac{1}{2}$. Therefore, the FFT method can waste valuable computer resources on unnecessary computations when the FDOA search range is relatively small. Another disadvantage is the fact that only integer values of k are evaluated in the FFT. In order to achieve higher resolution in this method, the argument in Equation (3-4) could be padded with zeros before the FFT is computed. This would effectively interpolate between integer values of k.

In order to compare the relative costs of the three methods, it is convenient to evaluate the number of floating point operations (flops), i.e., multiplies and adds, required for computation. In MATLAB, the "FFT" command is used to calculate the DFT. From [5], the approximate number of complex multiplies and adds required for one FFT (assumed to be radix-2 from here on) is

$$C_{cm-FFT} = \left(\frac{N}{2}\right)\log_2 N \tag{3-5}$$

$$C_{ca-FFT} = N\log_2 N, \tag{3-6}$$

where the subscripts *cm* and *ca* denote complex multiplies and complex adds, respectively. Now, since complex numbers are of the form (X + jY), multiplying two of them together requires four real multiplies (X_1*X_2, X_1*Y_2, Y_1*X_2, and Y_1*Y_2) plus two real adds (one to sum the real terms and one to sum the imaginary terms). Adding two complex numbers, on the other hand, requires just two real adds. Applying these two relations to Equations (3-5) and (3-6) establishes the number of real multiplies and real adds that occur during one FFT computation:

$$C_{rm-FFT} = 2N\log_2 N \tag{3-7}$$

$$C_{ra-FFT} = 3N\log_2 N, \tag{3-8}$$

where the subscripts *rm* and *ra* denote real multiplies and real adds, respectively. Given that the amounts of time required to execute a real multiply and a real add are roughly the same, the total number of flops required for one FFT computation is the sum of Equations (3-7) and (3-8):

$$C_{FFT} = 5N\log_2 N \tag{3-9}$$

Recalling that Equation (3-4) requires an FFT for every value of τ that is to be tested, the maximum cost in flops of the FFT method would be when the CAF is computed for all possible values of τ from $-N$ to N:

$$C_{MAX-FFT\,method} = 10N^2\log_2 N \tag{3-10}$$

Now, if the specific emitter-collector geometry and *a priori* knowledge of the signal parameters can narrow the range of τ values to be searched, the cost of the FFT method is reduced to:

$$C_{FFT\,method} = 10N_\tau N\log_2 N, \tag{3-11}$$

where N_τ is the total number of time lags for which the CAF will be computed. Equation (3-11) represents the cost metric for the FFT method that will be compared to the other two methods.

2. The Cross-Correlation Method

Equation (2-2) can also be rearranged such that it can be implemented with the cross-correlation function, which is defined as [4]:

$$R_{xy}(\tau) = \sum_{n=0}^{N-1} x(n)y^*(n+\tau), \tag{3-12}$$

where

$$R_{xy}(\tau) \equiv XCORR\left[x(n), y(n)\right] \tag{3-13}$$

The term “XCORR” is the MATLAB command that executes the cross-correlation function. Equations (2-2) and (3-12) share a common summation term, so the terms in the CAF expression must be rearranged and regrouped as follows:

$$CAF(\tau,k)=\sum_{n=0}^{N-1} s_1(n)\left[s_2(n+\tau)e^{+j2\pi\frac{k(n+\tau)}{N}}e^{-j2\pi\frac{k\tau}{N}}\right]^*, \qquad (3\text{-}14)$$

Note that the second, extra complex exponential is required in order to convert the *n* into the (*n* + *τ*) term in the first complex exponential. Realizing that $s_1(n)$ in Equation (3-14) is analogous to *x(n)* in Equations (3-12) and (3-13), and that $\left[s_2(n+\tau)e^{+j2\pi\frac{k(n+\tau)}{N}}e^{-j2\pi\frac{k\tau}{N}}\right]$ is analogous to *y(n* + *τ),* the CAF can be expressed as:

$$CAF(\tau,k)=XCORR\left\{s_1(n),\left[s_2(n)e^{+j2\pi\frac{k(n-\tau)}{N}}\right]\right\} \qquad (3\text{-}15)$$

Using Equation (3-15) to calculate the CAF for all values of *τ* and *k*, an individual XCORR computation is required for each value of *k*. The power of the XCORR function is that, for a given value of *k*, it calculates the values associated with *every* value of *τ*, from *–N* to *N*. This can be very desirable compared to the FFT method since the probable search range of TDOAs is likely to be much greater than the range of FDOAs that would need to be searched. Another advantage to this method is that *k* does not have to be an integer. In Equation (3-15), *k* can take on non-integer values, allowing for any desired degree of resolution in FDOA calculation. The main disadvantage with the XCORR method is that it is quite expensive since each invocation of XCORR requires more than three times the number of flops as an FFT.

In order to analyze its cost, the XCORR function can be broken down into FFTs. Note that the cross-correlation function, Equation (3-12) is essentially a convolution without the time reversal in the *y* term. Convolutions, and thus cross-correlations, can be computed efficiently by taking both signals into the frequency domain with FFTs, multiplying the result, and then performing an inverse FFT to get back to the time domain:

$$x(n) \underset{xcorr}{*} y(n) \ \equiv \ FFT^{-1}\left\{FFT\left[x(n)\right]FFT^{*}\left[y(n)\right]\right\} \tag{3-16}$$

Therefore, every XCORR function requires three FFT operations (at a cost of three times the number of flops listed in Equation (3-9)) plus N complex multiplies (or $6N$ flops) for the element-by-element multiplication of the two inner FFTs in Equation (3-16). The total number of flops required to compute a single XCORR function is therefore:

$$C_{XCORR} = 3*(5N\log_2 N) + 6N \ = \ 3N(2+5\log_2 N) \tag{3-17}$$

Now, assuming that k takes on the integer values in the range $-\frac{N}{2}+1$ to $\frac{N}{2}$, the maximum cost of the XCORR method would be approximately:

$$C_{MAX-XCORR\,method} = 3N^2(2+5\log_2 N) \tag{3-18}$$

In the likely event that geometry and signal parameters reduces the range of k values for which the CAF needs to be calculated, the actual cost for the XCORR method would be:

$$C_{XCORR\,method} = 3N_k N(2+5\log_2 N), \tag{3-19}$$

where N_k is the total number of frequency bins for which the CAF will be computed. Equation (3-19) represents the cost metric for the XCORR method that will be compared to the other two methods.

3. The Summation Method

For this final method of computing the CAF, Equation (2-2) is calculated directly. An advantage of the summation method is that it too can evaluate the CAF at any value of k, allowing for high resolution FDOA calculations. The disadvantage is that it requires a double loop to calculate the CAF for every value of τ and k. The reliance on loops is very costly, particularly for interpretive programming languages such as MATLAB.

The maximum cost for the summation method would be for the case where the CAF is computed for all $2N$ values of τ and all N values of k (assuming just the integer values of k). Assuming that the cost of the conjugation operation is negligible, and that the multiplies in the complex exponential are done ahead of time, each iteration of the summation will require two complex multiplications, or 12 flops. Since the summation goes through N iterations, the summation method's maximum cost in flops is:

$$C_{MAX-SUM\ method} = N*12*2N*N = 24N^3 \tag{3-20}$$

Now, assuming that the range of τ and k values can be narrowed down, the actual cost of the summation method would be:

$$C_{SUM\ method} = 12N_\tau N_k N, \tag{3-21}$$

where N_τ and N_k are the total numbers of τ and k values, respectively. Equation (3-21) can be used to compare costs with the other two methods. Equations (3-11), (3-19), and (3-21) can be used to evaluate which method would be most efficient for a particular emitter-collector geometry and set of signal parameters, which of course would determine the range of τ and k values (and thus N_τ and N_k) for which the CAF would need to be evaluated.

Table (3-1) summarizes the maximum and narrowed search range complexities of the three methods described above.

Method	Maximum Complexity (flops)	Narrowed Search Range Complexity (flops)
FFT	$10N^2 \log_2 N$	$10N_\tau N \log_2 N$
Cross-Correlation	$3N^2(2+5\log_2 N)$	$3N_k N(2+5\log_2 N)$
Summation	$24N^3$	$12N_\tau N_k N$

Table 3-1. Computational Complexities of Three Direct CAF Computation Methods.

By comparing the maximum complexities in Table (3-1), if all possible integer values of k and τ were to be searched, then the FFT method would be the most efficient. If the range of τ and k values were narrowed by geometric and signal parameter considerations, however, the most efficient method would depend upon the total number of possible TDOAs and FDOAs, N_τ and N_k, that would be evaluated.

It is important to note that the three methods described represent direct computations of the CAF. In most cases, N_τ and/or N_k would be large enough to make "brute-force" computation of the CAF (using any of the three methods) an overwhelming burden on computer resources. A more efficient approach involves a two-step computation of the CAF, implementing a "coarse mode" and a "fine mode" to compute the TDOA and FDOA within reasonable accuracy. [1] This is the approach implemented by the MATLAB code in Appendix A. The following section describes the approach in detail.

B. ANALYSIS OF CAF SOFTWARE

As mentioned in the section above, the three methods of directly computing the CAF are computationally much too expensive to use as a one-step process, even when the number of τ and k values is narrowed due to knowledge of the specific geometry in use. In order to reduce the processing burden to an acceptable level, computing the CAF can be broken into two distinct parts: a "coarse" mode and a "fine" mode. In the coarse mode, all possible values of τ and k (as determined by geometry and signal parameters) are processed in order to produce a rough (or coarse) estimation of the TDOA and FDOA between the two signals. The coarse estimates are then fed into the fine mode, which computes the final TDOA and FDOA calculations. An algorithm for the coarse mode is described in [1] and is the basis for the code generated for this thesis. As for the fine mode, the summation method described in the previous section is used. The following subsections describe the coarse and fine modes, as well as their implementation in "CAF.m," which is listed in Appendix A.

1. The Coarse Mode

Reference [1] provides an algorithm to calculate coarse estimates of the TDOA and FDOA between two signals. The goal of the algorithm is to produce coarse estimates that are accurate enough to enter a fine mode, while keeping processing burden as small as possible. In order to accomplish this, the algorithm makes use of convolution properties, as well as breaking the input signals into smaller blocks to speed processing. The algorithm is represented by the following modified version of Equation (2-2) [1]:

$$CAF_R(qN_1 + m, v) = \sum_{k=0}^{N_1-1} S_1(k+v;R)S_2^*(k;R+q)e^{-j2\pi\frac{km}{N_1}} \qquad m = 0,\ldots,\frac{N_1}{2} \tag{3-22}$$

In Equation (3-22), $S_1(k;R)$ refers to the FFT of the Rth block of $s_1(n)$ and $S_2(k;R)$ refers to the FFT of the Rth block of $s_2(n)$. As mentioned, S_1 and S_2 are processed in sub-blocks that are smaller than the total number of data points in each signal, *N*. The size of each sub-block is N_1 elements, *q* is an index for the sub-block(s) being processed, and *v* represents the frequency bin shift. The notation CAF_R refers to the calculation which combines the *R*th block of S_1 with the (*R* + *q*)th block of S_2. In order to avoid circular convolution effects, 50 percent overlap is utilized, such that the (*R* + *q*)th block of S_2 consists of $\frac{N_1}{2}$ data elements and $\frac{N_1}{2}$ zeros. Recalling Equation (3-4), Equation (3-22) can be rewritten as:

$$CAF_R(qN_1 + m, v) = DFT\left[S_1(k+v;R)S_2^*(k;R+q)\right] \tag{3-23}$$

Equation (3-23) calculates CAF_R for all values of *m* (from 0 to $\frac{N_1}{2}$) for a given *q* and *v*. For every fixed combination of *q* and *v*, CAF_R is computed for each sub-block (i.e., for *R* = 1 to $\frac{N}{N_1}$). Figure (3-1) illustrates how the sub-blocks would be processed for two signals of length *N* = 4096 and a sub-block length of N_1 = 2048. For *R* = 1, the first block of S_1 is processed with the first and second blocks of S_2 (*q* goes from 0 to 1, making (*R* + *q*) go from 1 to 2). For *R* = 2, the second block of S_1 is processed only with the second block of sub-block S_2 (in this case, *q* cannot exceed 0 since (*R* + *q*) cannot exceed the total number of sub-blocks). The magnitudes of the calculations are then averaged to obtain one value for every fixed *q* and *v*. For example, in Figure (3-1), there are two computations for which *q* = 0. These two magnitudes are averaged to get the

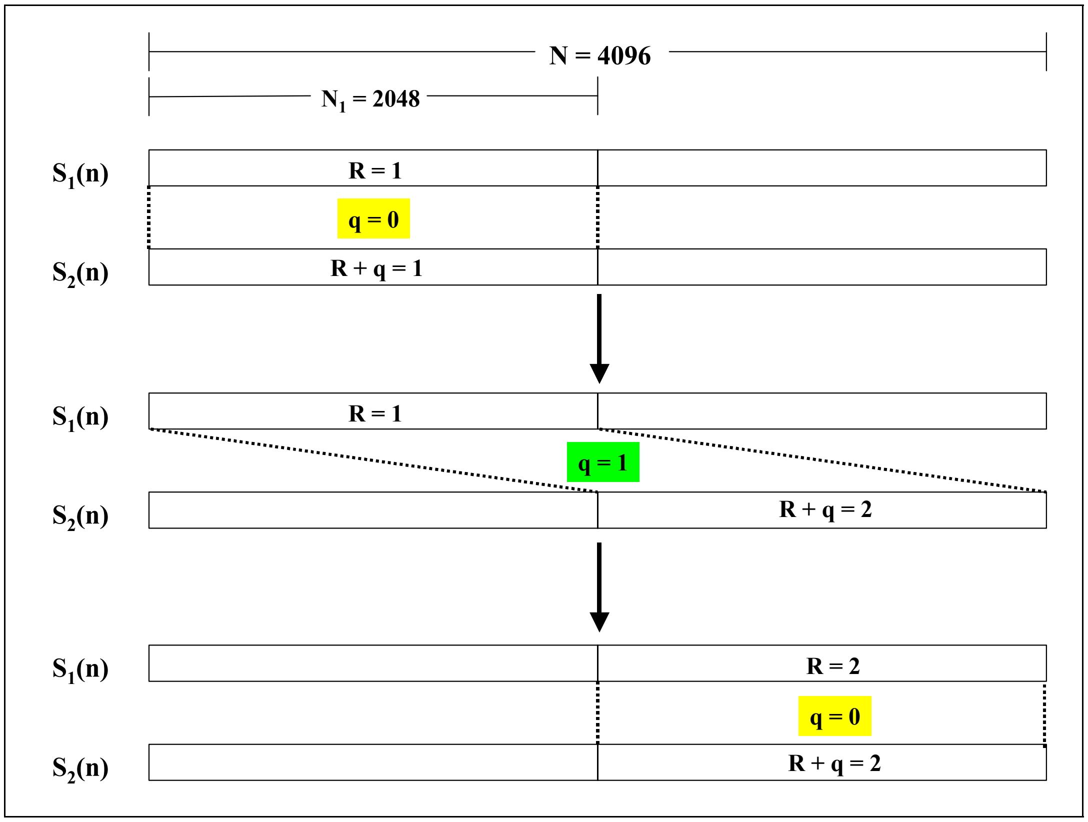


Figure 3-1. Coarse Mode Sub-Block Processing.

value associated with q = 0. There is only one computation for which q = 1, so that magnitude is the one associated with q = 1. Finally, the maximum of all the averaged values is determined, along with the values of q, v, and m that produced the peak. With these three parameters, the coarse TDOA and FDOA are computed as follows:

$$TDOA_{coarse} = qN_1 + m \quad (in\,samples) \tag{3-24}$$

$$FDOA_{coarse} = v\frac{N}{N_1} \quad (freqency\,bin\,\#) \tag{3-25}$$

The coarse values for TDOA and FDOA obtained from Equations (3-24) and (3-25) are then used as the starting point for the fine mode computations.

A major limitation of this algorithm is that it does not work properly for situations where the TDOA is a negative value. This is easily overcome, however, by simply reversing the order of the input signals. In other words, S_1 and S_2 in Equation (3-23) can be switched if necessary to avoid a negative TDOA. The only effect on TDOA and FDOA when reversing the order of the signals is that the sign is flipped in both cases. Unlike the case for negative TDOAs, the algorithm works fine for both positive and negative FDOAs.

2. The Fine Mode

Once coarse estimates of the TDOA and FDOA have been computed by finding the maximum value of the CAF, it is necessary to interpolate that peak in order to obtain the actual TDOA and FDOA within the desired resolution. This is what the fine mode accomplishes. Since the fine mode need only evaluate a few TDOAs and FDOAs on either side of the coarse estimates, one of the three direct computation methods described in section A above can be utilized without much burden on processing resources.

In order to determine which of the three methods is most efficient for fine mode calculations, it is convenient to compare the "Narrowed Search Range" computational complexity equations summarized in Table (3-1). Since N_τ and N_k will be relatively small in the fine mode, it is clear that the FFT method will be more efficient than the cross-correlation method. So, to decide which method to use, the FFT and summation equations can be compared as follows:

$$12N_\tau N_k N < 10N_\tau N \log_2 N \tag{3-26}$$

Canceling like terms and simplifying yields:

$$N_k < \frac{5}{6}\log_2 N \tag{3-27}$$

or

$$N > 2^{1.2N_k} \tag{3-28}$$

If Inequalities (3-27) and (3-28) are true, then the summation method is the one to use. Otherwise, the FFT method would be the most efficient. It is a fair assumption that N_k will not be more than 10 for fine mode calculations. Therefore, the summation method would be the most efficient method when $N > 2^{1.2(10)}$ *or* $N > 4096$. For coding purposes, the assumption was made that sampled signals used in the CAF would contain more than 4096 elements. Therefore, the fine mode is accomplished by implementing the summation method. The MATLAB program written to perform the coarse and fine computations is called "CAF.m". The next section describes it in detail.

3. The "CAF.m" Program

The program CAF.m, listed in Appendix A, is a MATLAB function that computes the TDOA and FDOA between two sampled signals. It is invoked in the MATLAB command window with a line of the form:

[TDOA, FDOA] = CAF(S1, S2, Max_f, fs, Max_t);

The input arguments *S1* and *S2* are the two sampled signals in *analytic signal* format. *Max_f* is the maximum magnitude of FDOA, in Hertz, that is expected between the two signals. The argument *fs* is the sampling frequency used to generated the sampled signals *S1* and *S2*. The sampling frequency is assumed to be the same for both signals. *Max_t* is the maximum TDOA, in seconds, expected. Note that *Max_f* and *Max_t* are functions of the geometry and signal parameters for a given scenario. Also note that *Max_t* must be positive do to the coarse mode algorithm's constraint, as described in section 1 above. If the expected *Max_t* is negative, then *S1* and *S2* need only be reversed in the function call shown above (e.g., *[TDOA, FDOA] = CAF(S2, S1, Max_f, fs, Max_t);*). The output arguments *TDOA* and *FDOA* make the computations available to the MATLAB user in variables of the same names.

The first step in CAF.m is to reshape *S1* and *S2* to ensure that they are column vectors. This takes advantage of the fact that MATLAB stores variables and performs computations on them in a column-wise fashion. Next, the most appropriate size of sub-block (*N1*) is determined. *N1* nominally starts out at 1024, but the while loop ensures that *N1* is large enough to ensure proper resolution. For example, if the maximum frequency bin expected $\left(\frac{Max\_f}{fs} N1 \right)$ were less than one, then the resolution would not be good enough to discern the correct frequency bin. Until acceptable resolution is obtained, *N1* is successively multiplied by two. This takes advantage of MATLAB's more efficient FFT operations on vectors whos sizes are powers of two. In no case will *N1* be larger than $2^{19} = 524288$, as this is roughly the maximum size for which processing is efficiently possible. Clearly this is dependent upon the specific system being used for the processing. In some cases, the sub-block size may be larger than the size (N) of the signal vectors. In this case, CAF.m will pad the signal vectors with enough zeros to make the overall length equal to *N1*.

Next, CAF.m determines the total number of sub-blocks that are in the signal vectors, *Number_of_Blocks*. This is simply $\frac{N}{N_1}$, where N is the length of the signals and N_1 is the size of one sub-block. Every block of S_1 will be processed, so the variable R will go from one to *Number_of_Blocks*. The program then uses the input arguments *Max_t* and *Max_f* to determine the range of values for q and v, respectively, that must be used in the subsequent calculations. Note that each sub-block represents a period of time equal to $N_1 T_s$, and q represents multiples of this value. Therefore, q's values will be dependent upon how large the maximum expected TDOA (*Max_t*) is. For example, if *Max_t* is three microseconds and $N_1 T_s$ is two microseconds, then q need only take on the values zero, one, and two. Any value greater than two would cause unnecessary processing since it would correspond to TDOAs above *Max_t*. Likewise, the frequency bin values, v, that need to be computed are determined from the user's defined *Max_f*.

The heart of the coarse mode section of CAF.m is a triple nested loop, which runs through all of the required values for R, q, and v. Figure (3-2) is a flow chart that shows the triple loop and the processing steps that occur for the coarse mode. The outermost loop runs through all of the required values of v. The next loop runs through all values of R (i.e., from one to *Number_of_Blocks*). Within the R loop, the program picks out the elements of *S1* that correspond to the Rth block, and then performs an FFT on the result. As required by Equation (3-23), the resulting FFT is then shifted by v frequency bins. The innermost loop then runs through all required values of q. As per Equation (3-23), the (R + q)th block of *S2* is then obtained and subjected to an FFT. Note that, as required by the algorithm, only the first $\frac{N1}{2}$ data elements of the (R + q)th block are used, with the

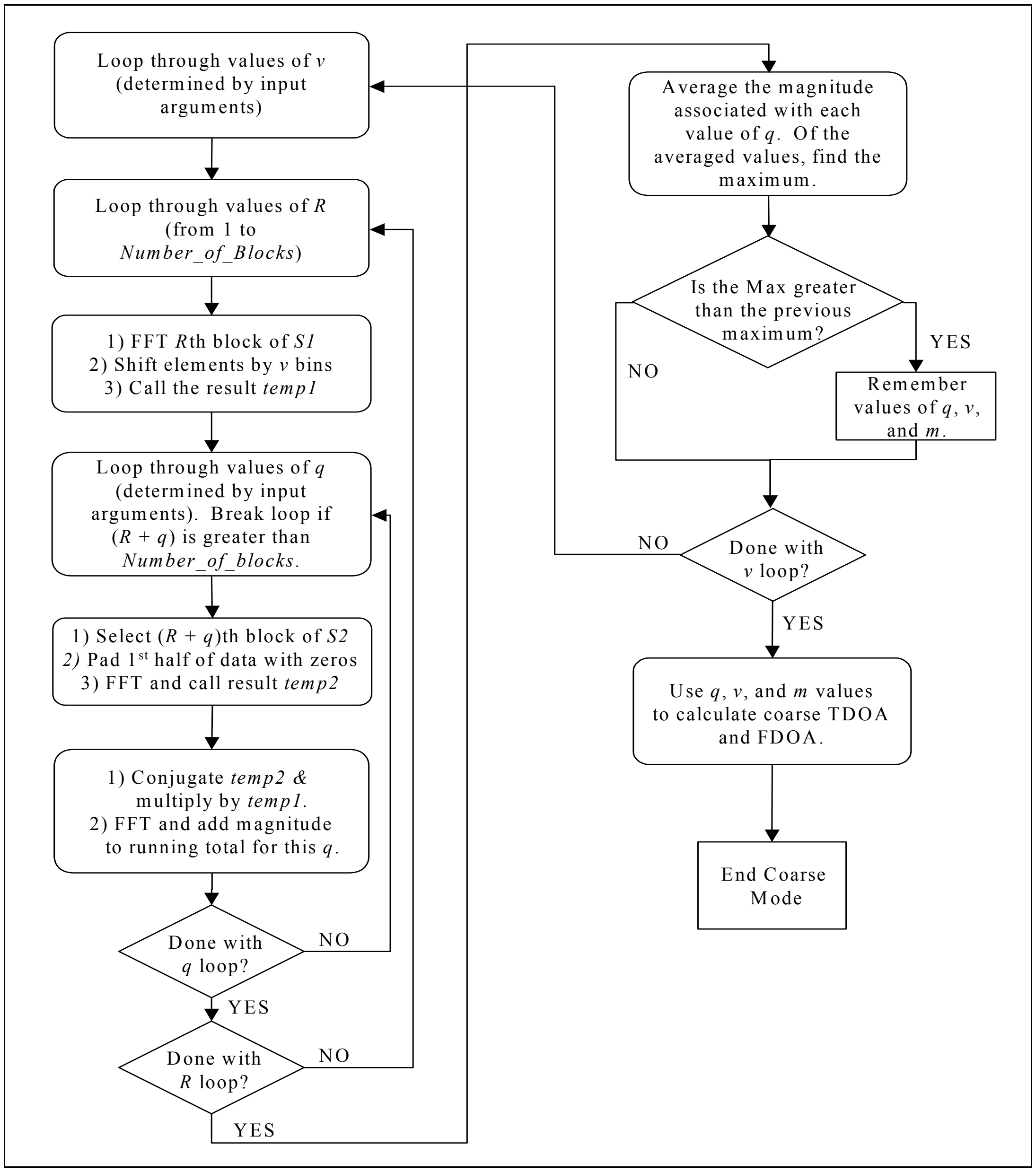

Loop through values of v (determined by input arguments)
Loop through values of R (from 1 to Number_of_Blocks)
1) FFT Rth block of S1
2) Shift elements by v bins
3) Call the result temp1
Loop through values of q (determined by input arguments). Break loop if (R + q) is greater than Number_of_blocks.
1) Select (R + q)th block of S2
2) Pad 1st half of data with zeros
3) FFT and call result temp2
1) Conjugate temp2 & multiply by temp1.
2) FFT and add magnitude to running total for this q.
Done with q loop?
NO
YES
Done with R loop?
NO
YES
Average the magnitude associated with each value of q. Of the averaged values, find the maximum.
Is the Max greater than the previous maximum?
YES
NO
Remember values of q, v, and m.
Done with v loop?
NO
YES
Use q, v, and m values to calculate coarse TDOA and FDOA.
End Coarse Mode

Figure 3-2. Flow Chart of Coarse Mode in CAF.m.

remaining $\frac{N1}{2}$ elements being all zeros. A final FFT is performed on the product of the *S1* block and the conjugated *S2* block. The magnitude of the result is then added to the accumulating total for all previous instances of that particular value of *q*. Once the *q* and *R* loops are finished, the magnitudes are divided by the total number of times that each value of *q* was used. This calculates the averages, as required by the algorithm. Next, the maximum value contained in the resulting matrix of values is compared to the current max value. If it is greater, then the program saves that value, as well as the *q*, *v*, and *m* that caused the new maximum. Once the *v* loop is completed, all computations have been accomplished for the coarse mode, and the resulting values of *q*, *v*, and *m* are then used to compute the coarse TDOA and FDOA using Equations (3-24) and (3-25), respectively. Note that in Equation (3-24), *m* represents the frequency bin number of the N1-sized FFT. In CAF.m, *m* is really the *index* into the FFT. In order to convert that index into the actual frequency bin number, the term $\left(-\frac{N1}{2}+1\right)+m$ is used since the FFT elements begin with the $\left(-\frac{N1}{2}+1\right)$th frequency bin. So, to summarize, the $\left(-\frac{N1}{2}+1\right)+m$ term in CAF.m is equivalent to *m* in Equation (3-24).

The next section of CAF.m represents the fine mode of the CAF computation. Because the accuracy of the course estimations is not great, and because noise in the input signals degrades the accuracy even further, the initial fine computations are made for a fairly large number of parameter values. The set of time samples computed (contained in the vector *tau_val*) is the coarse TDOA estimation plus or minus 10 samples. Likewise, the set of frequency bins computed (contained in the vector *k_val*) is the coarse FDOA estimation plus or minus 10 bins.

Next, the summation method (Equation (2-2)) is used to carry out the fine calculations. This requires a double loop to run through all of the values contained in the *k_val* and *tau_val* vectors. For each value of *k*, the complex exponential term is computed as a vector, so that *N* separate calculations are not required within the inner loop. This reduces the overall processing burden. In the inner loop, the *S2* vector (which is already conjugated as required in Equation (2-2)) is shifted the appropriate number of time samples. The shift operation is accomplished by the MATLAB function shiftud.m, obtained from [9] and listed in Appendix A. Finally, the *S1*, S2, and *exponents* vectors are multiplied (element by element in one step using MATLAB's ".*" command) and then summed to obtain one scalar value. The magnitude of that value is then stored in a matrix for that particular value of *k* and *t*. Once the double loop is finished, the maximum value in the CAF matrix is determined, along with the values of TDOA and

FDOA that caused that maximum value. The variables *TDOA* and *FDOA* then contain the initial fine mode calculations.

The TDOA is displayed to the user in both samples and seconds, and the FDOA is displayed in both digital frequency and Hertz. The user is also told to what resolution the solutions are calcualated, in seconds for the TDOA and in Hertz for the FDOA. The user is then given the following options:

1) Re-compute with finer resolution for TDOA

2) Re-compute with a finer resolution for FDOA

3) Re-compute with finer resolutions for both TDOA and FDOA

4) Keep the current solutions

The TDOA computation involves shifting the *S2* vector a specific number of samples (or elements). Since it is only possible to shift elements by an integer amount, the only way to increase the TDOA resolution is to increase the sampling frequency of the signal vectors. This follows from the fact that the TDOA in seconds is equal to the TDOA in samples divided by the sampling frequency. A very quick way to increase the sampling frequency of a vector in MATLAB is to use the built-in *interp* function, which will resample a vector at a specified integer multiple of the original sampling frequency. The resulting vector's length is the specified integer times the original length, thereby ensuring that the exact same period of time is covered in the new vector. In CAF.m, if the user chooses to compute a TDOA with higher resolution, then *S1* and *S2* are resampled at twice their sampling frequency, thereby increasing the TDOA resolution by a factor of two. Now, for a fine TDOA computation, the true value will be within 0.5 samples on either side of the calculation. Therefore, successive TDOA computations (i.e., after doubling the sampling frequency) need only check three possible TDOAs: the previously calculated TDOA multiplied by two, plus the value on either side of it. For example, if a TDOA is computed to be 18 samples, the true value would be somewhere between 17.5 and 18.5 samples. Doubling the sampling frequency, the TDOAs to check would be $18*2 \pm 1$, or sample numbers 35, 36, and 37.

The FDOA computation simply involves the value of *k* used in the complex exponential term in Equation (2-2). As mentioned in section III.A.3 above, an advantage of the summation method is that any value of *k* can be used; it is not restricted to integer numbers. This makes increasing FDOA resolution quite easy. The approach used in CAF.m is to increase the FDOA resolution by a factor of 10. For a fine FDOA computation, the true value will be in the range of $k \pm 0.5x10^{x}$, where *x* is the exponent when *k* is in scientific notation form. Taking 11 equally spaced values in that range will provide a resolution improved by a factor of 10. For example, if a fine FDOA is computed to be 0.6 ($6x10^{-1}$) bins, the true value will be somewhere in the range $0.6 \pm 0.5x10^{-2}$, or 0.55 to 0.65. Therefore, the FDOA would be recomputed by testing the 11 values of *k* from 0.55 to 0.65, spaced 0.01 apart.

The entire fine mode in CAF.m is enclosed in a while loop that continues to perform more improved calculations of TDOA and/or FDOA until either: 1) the user is satisfied with the results, or 2) the maximum processing capacity has been reached. In the second case, TDOA improvement ends when the length of *S1* and *S2* reaches $2^{19} = 524288$. The FDOA can continue to be improved upon until the user is satisfied. When processing capacity is reached, the options that include TDOA optimization (numbers one and three in the list above) are removed from the user's list of options. Once the optimization is complete and a final TDOA and FDOA have been reached, the user is given the option of displaying the actual CAF surface graphically. The CAF surface is computed and plotted by the CAF_peak.m function, which is listed in Appendix A, and described in the next section. The CAF surface is computed for the original *S1* and *S2*, in order to minimize processing burden. The surface is computed for the TDOA ± 50 samples, and for the FDOA ± 20 frequency bins. It is important to note that the CAF surface is for display only, as its peak occurs at un-interpolated values of TDOA and FDOA. Once the surface is plotted, or if the user opts to not plot it, the CAF.m function is completed. Section III.C below shows the results of running some example signal sets through CAF.m.

4. The "CAF_peak.m" Program

The program CAF_peak.m, listed in Appendix A, is a MATLAB function that computes the CAF surface by comparing two sampled signals. It is invoked with a line of the form:

```
[TDOA, FDOA, MaxAmb, Amb] =
                    CAF_peak(S1, S2, Tau_Lo, Tau_Hi, Freq_Lo, Freq_Hi, fs);
```

The input arguments *S1* and *S2* are the two sampled signal vectors in *analytic signal* format. The arguments *Tau_Lo* and *Tau_Hi* represent the lowest and highest number of samples for which to compute the CAF surface. Likewise, *Freq_Lo* and *Freq_Hi* represent the lowest and highest digital frequencies for which to compute the CAF surface. Finally, *fs* is the sampling frequency. The output arguments *TDOA* and *FDOA* make the computations available to the MATLAB user in variables of the same names. Note again that this program does not compute interpolated solutions of TDOA and FDOA. It uses the FFT method described in section III.A.1 above. The TDOA's resolution is therefore only 0.5 samples, or $0.5T_s$ seconds. The FDOA's resolution is $\frac{0.5}{N}$ (digital frequency), or $\frac{0.5}{N}f_s$ Hertz. The output arguments *MaxAmb* and *Amb* return the magnitude of the surface's peak and the matrix of values for the CAF surface (as bounded by the input arguments), respectively.

The first section of CAF_peak.m performs a number of checks to ensure that the input arguments are valid. These checks are not really necessary when CAF.m calls the function, because CAF.m properly calculates the input arguments. But CAF_peak.m can also be called directly by a user in the MATLAB command window. This is where the checks become useful. The function checks to ensure that there are enough input arguments, and that *S1* and *S2* are indeed vectors (and not matrices). The program then reshapes the signal vectors to ensure that they are columns in order to take advantage of MATLAB's column-wise nature. The program uses zero padding to ensure that the signals are of the same length, and that their length is a power of two (again, for computing efficiency). Next, *Tau_Lo* and *Tau_Hi* are checked to ensure that they are integers in the range –*N* to *N* and *Freq_Lo* and *Freq_Hi* are checked to ensure that they are in the range –0.5 to 0.5. Finally, the program ensures that *Tau_Lo* and *Freq_Lo* are in fact smaller than *Tau_Hi* and *Freq_Hi*, respectively.

Since the FFT method computes the CAF for *all* frequency values represented by the bins $k = \left(-\frac{N}{2}+1\right)$ to $\frac{N}{2}$, the program must determine the indices into each FFT that correspond to the user's defined range of *Freq_Lo* to *Freq_Hi*. Next, the CAF is computed in a for loop that runs through all of the values defined by the user's range of *Tau_Lo* to *Tau_Hi*. For each value, Equation (3-4) is computed by performing an FFT on the product of *S1* with the conjugated *S2*, which is shifted by an amount equal to the loop variable *t*. The appropriate values are then extracted using the previously calculated indices. The magnitude of the resulting vector is then placed as a new column in the *Amb* matrix. When the loop is completed, *Amb* contains all values for the CAF surface, as bounded by the input arguments. Furthermore, *Amb*'s rows represent frequency bins and its columns represent numbers of samples. The maximum value of *Amb* is the peak of the CAF surface, and the row and column associated with that peak are the FDOA and TDOA, respectively. Finally, CAF_peak.m produces four different graphical views of the CAF surface. The first is a three-dimensional view, the second is a two-dimensional view looking at the TDOA axis, the third is a two-dimensional view along the FDOA axis, and the final plot is a two-dimensional flat view looking down on the surface. The next section shows the result of running some example signal sets through the CAF.m and CAF_peak.m programs.

C. EXAMPLES AND RESULTS

In order to test and evaluate the CAF.m and CAF_peak.m programs to ensure that they performed accurate computations, signal pairs with *known* TDOAs and FDOAs embedded in them were required. To aid in the testing and evaluation, a signal generation software package from Statistical Signal Processing, Inc. (SSPI) [10] was used to create signals with TDOAs and FDOAs. The SSPI software was capable of producing only *constant* TDOAs and FDOAs, which caused no problem for testing and evaluation purposes. But as discussed in Chapter I, constant TDOAs and FDOAs are not found in real-world applications. Emitter-collector geometries change with time due to relative motion between them, making the associated TDOAs and FDOAs themselves time-varying. This issue is discussed further in Chapters IV and V.

To validate the accuracy of the CAF.m and CAF_peak.m programs, several sampled signals with different time delays and frequency offsets were generated with SSPI software. Several combinations of signals were input into CAF.m and CAF_peak.m to ensure that their solutions matched the known TDOAs and FDOAs. The following subsections detail the results.

1. Constant TDOA With Zero FDOA

For Case #1, a pair of signals with the following parameters were input into the CAF.m program:

Signal Type: BPSK with rectangular envelope

Carrier Frequency: 0.21 (digital frequency)

Samples Per Bit: 16

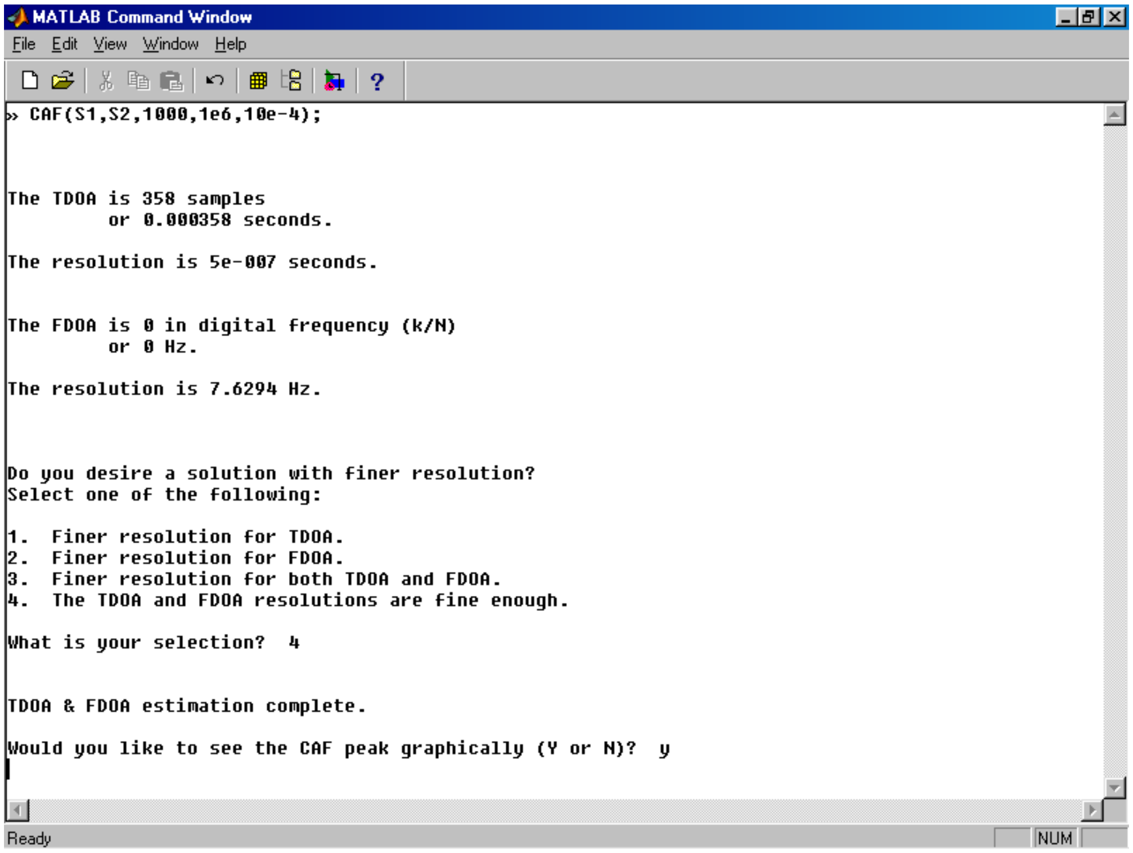
Signal-to-Noise Ratio for the two signals: 20 dB & 20 dB

Number of Samples: 65536

TDOA: 358 samples

FDOA: 0 (digital frequency)

Note that digital frequency is used and no specific sampling frequency is defined. For testing purposes, however, a sampling frequency of f_s = 1 MHz was assumed. This makes the effective symbol rate $\frac{1MHz}{16\,samples/bit} = 62{,}500\,bps$. The expected TDOA is then $\frac{358\,samples}{f_s} = \frac{358}{1x10^6} = 3.58x10^{-4}$ seconds. The expected FDOA is 0 * f_s = 0 Hz. Figure (3-3) shows the MATLAB command window with the results of running CAF.m on the signal pair described above. Note that because the signals were generated with zero FDOA and an integer number of samples for TDOA, CAF.m's first computation



```
» CAF(S1,S2,1000,1e6,10e-4);



The TDOA is 358 samples
         or 0.000358 seconds.

The resolution is 5e-007 seconds.


The FDOA is 0 in digital frequency (k/N)
         or 0 Hz.

The resolution is 7.6294 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 3-3. CAF.m Results – Case #1.

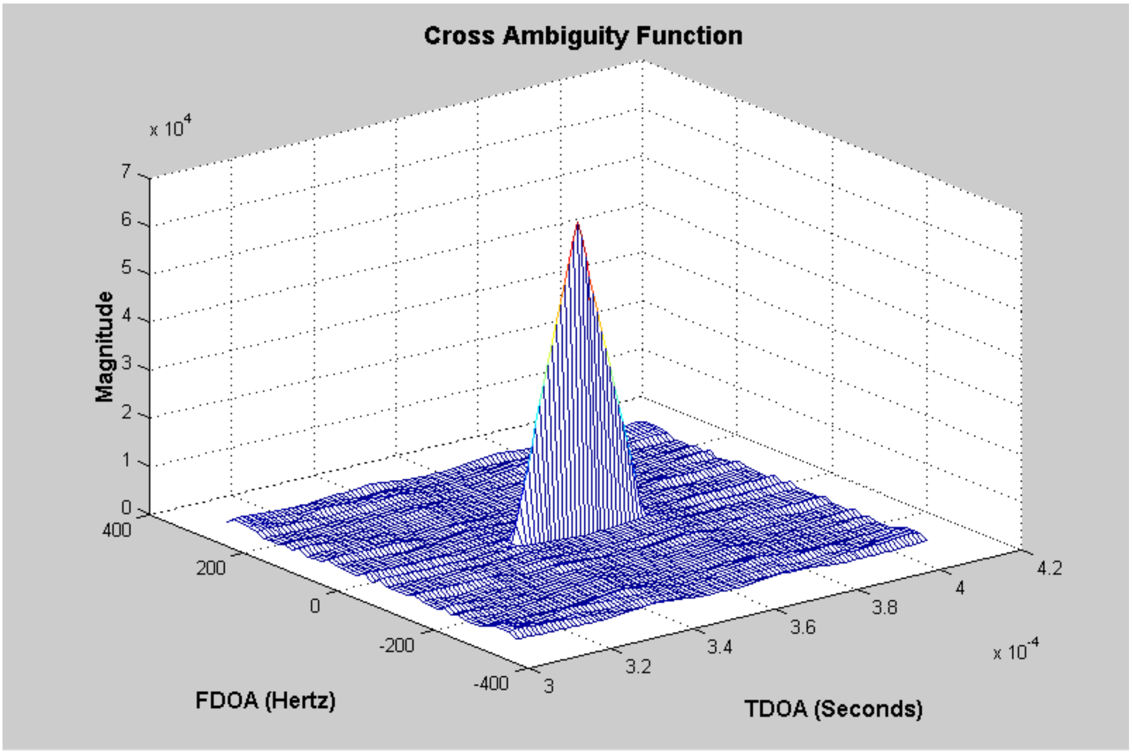


Figure 3-4. 3-D CAF Surface – Case #1.

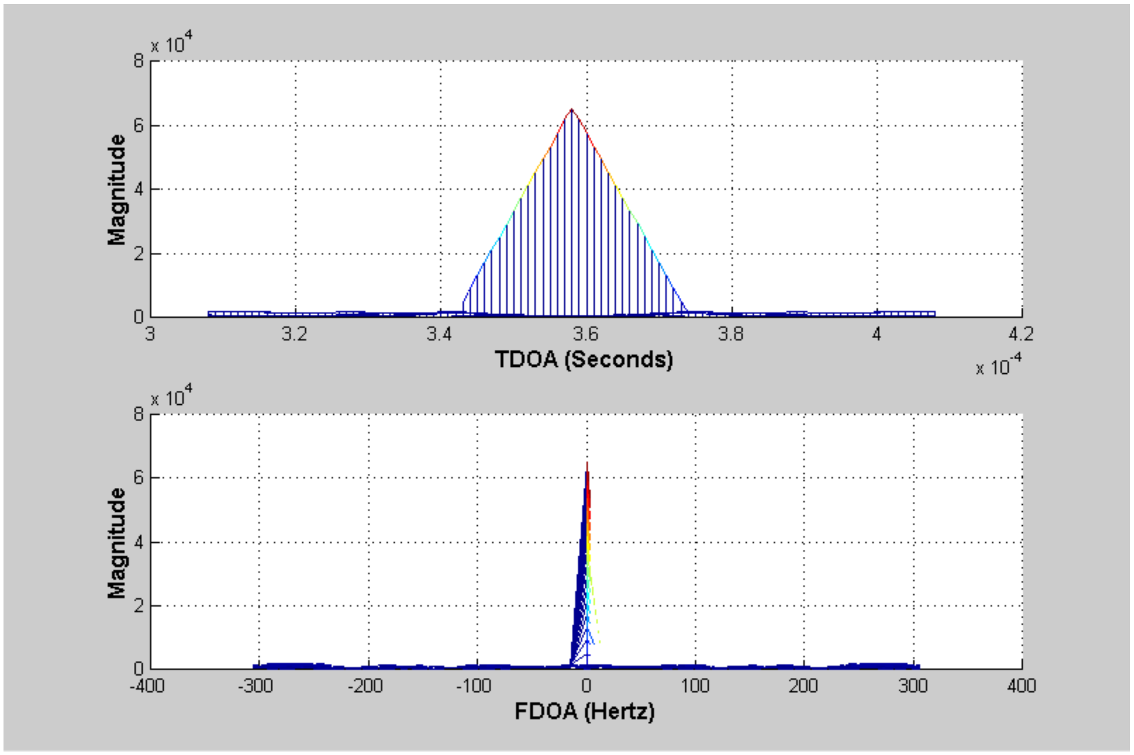
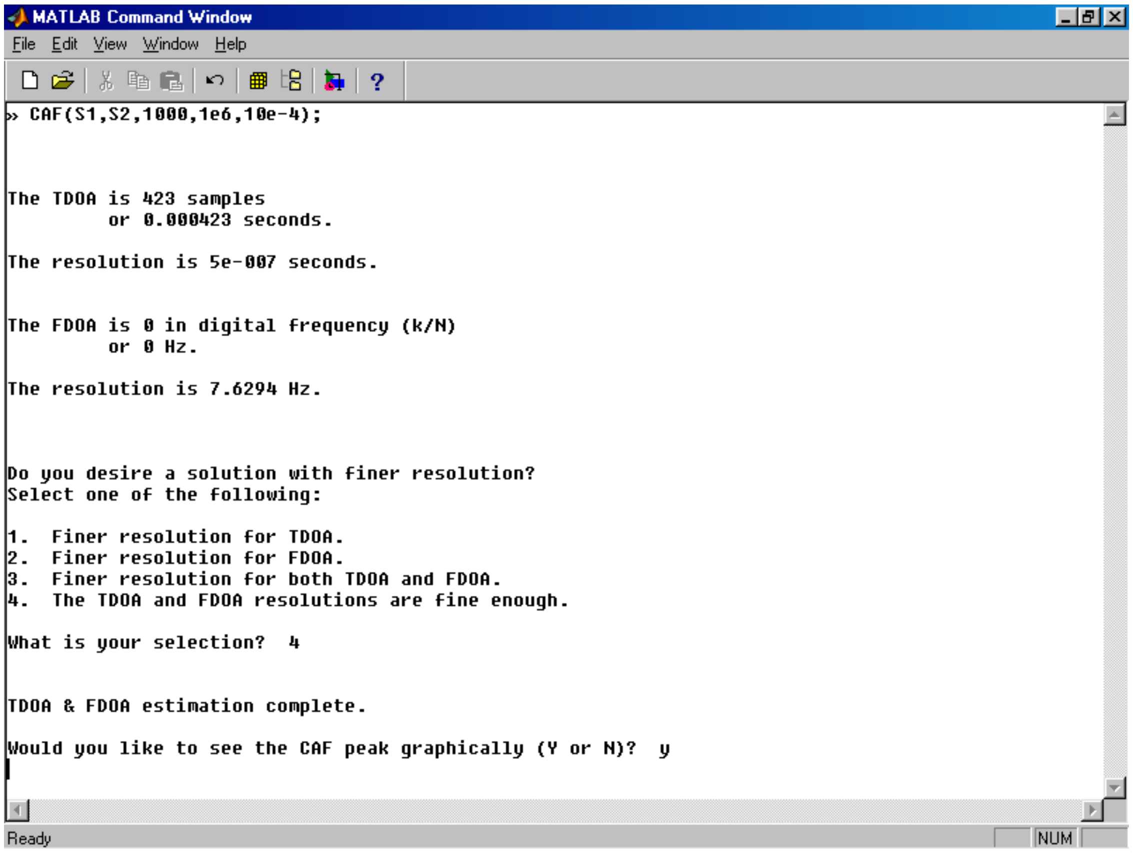


Figure 3-5. 2-D Cuts Through CAF Surface – Case #1.

produces the *exact* solution. Therefore, no further iterations of the fine mode were required. Figure (3-4) shows a three-dimensional view of the CAF surface, while Figure (3-5) provides two-dimensional slices of the surface along the TDOA and FDOA axes. Note the triangular shape of the surface along the TDOA axis. This makes sense considering that the basic CAF equation is in the form of a convolution summation. When two rectangular-envelope pulses are convolved, the result is triangular.

For Case #2, a pair of signals with the following parameters were input into the CAF.m program:

Signal Type:	BPSK with half-cosine envelope
Carrier Frequency:	0.21 (digital frequency)
Samples Per Bit:	16
Signal-to-Noise Ratio for the two signals:	0 dB & -20 dB
Number of Samples:	65536



```
» CAF(S1,S2,1000,1e6,10e-4);



The TDOA is 423 samples
         or 0.000423 seconds.

The resolution is 5e-007 seconds.


The FDOA is 0 in digital frequency (k/N)
         or 0 Hz.

The resolution is 7.6294 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 3-6. CAF.m Results – Case #2.

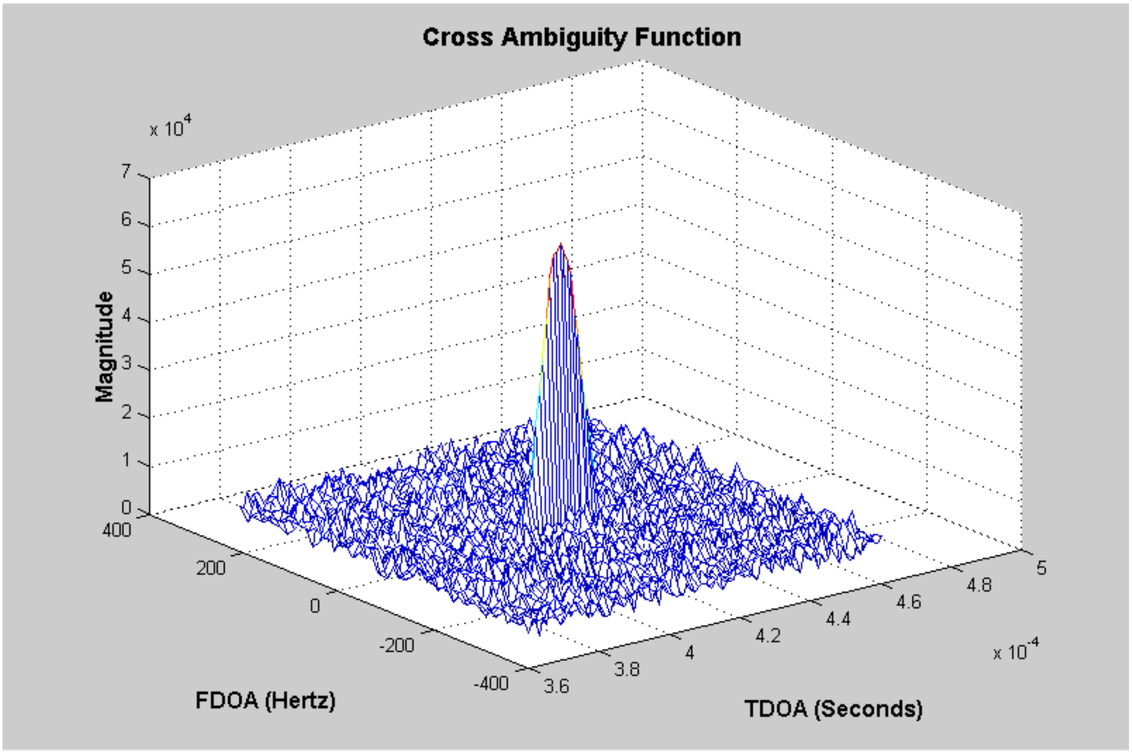


Figure 3-7. 3-D CAF Surface – Case #2.

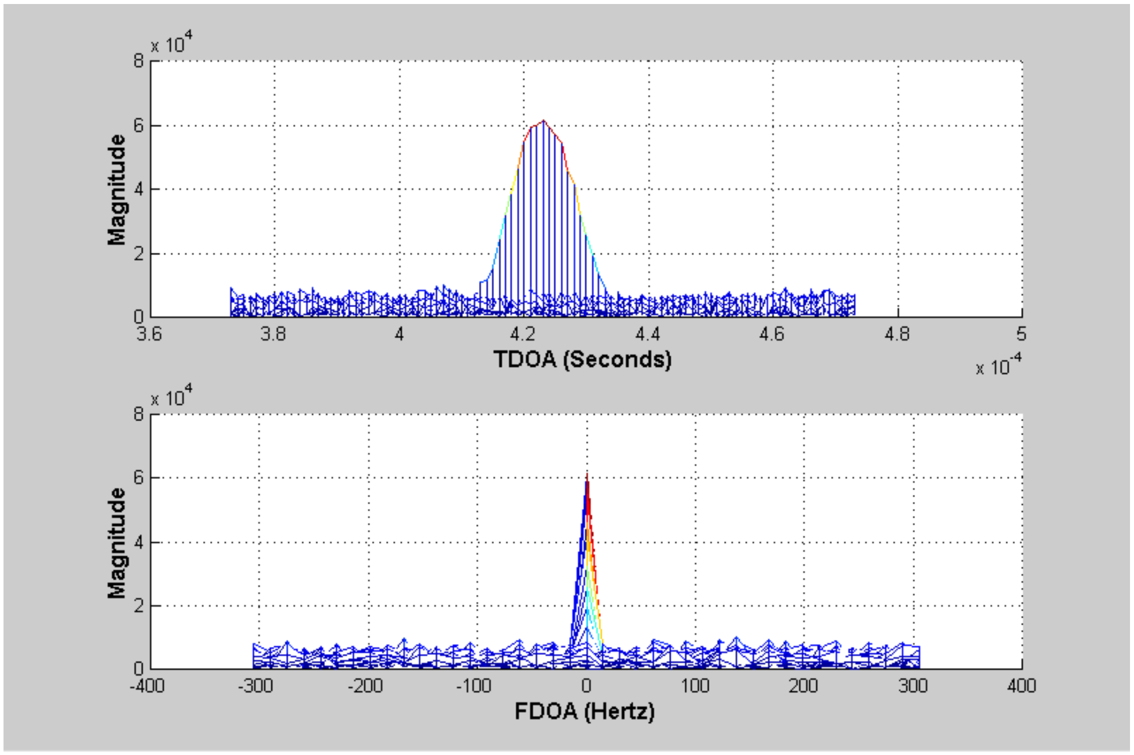


Figure 3-8. 2-D Cuts Through CAF Surface – Case #2.

TDOA: 423 samples

FDOA: 0 (digital frequency)

Note that there are three main differences between Case #2 and Case #1. First, the signals have a half-cosine envelope rather than a rectangular one. Second, the signals have more noise, as seen in their smaller SNRs. Third, the expected TDOA is different: 423 samples or $4.23x10^{-4}$ seconds (f_s = 1 MHz). Figures (3-6) through (3-8) show the results of running this signal set through CAF.m. Again, note that CAF.m computed the exact solutions since the actual TDOA was an integer number of samples and the FDOA was zero. Also note the effect of the lower SNRs in this pair of signals. The noise floor around the CAF peak is significantly higher than in Case #1. Finally, note the shape of the surface along the TDOA axis. It is more sinusoidal, or perhaps Gaussian, in shape. This makes sense since the signals' envelopes were half-cosines. The convolution of two sinusoidal shapes gives a similar shape. In the next subsection, signal pairs with non-zero TDOAs and FDOAs will be examined.

2. Constant TDOA and Constant FDOA

The next two pairs of signals have constant TDOAs and FDOAs embedded within them. As stated before, constant TDOAs and FDOAs are unrealistic for real-world geometries. Also, it is impossible to have simultaneously constant TDOAs and FDOAs. This is because whenever a constant TDOA exists, the FDOA must always be zero! After all, geometries that produce constant TDOAs are such that the individual Doppler shifts between each collector and the emitter are identical. The difference between the Dopplers, the FDOA, is therefore zero! Using signals with constant TDOAs and FDOAs is, however, quite convenient for testing the operation of the CAF software. For Case #3, a pair of signals with the following parameters were input into the CAF.m program:

Signal Type: BPSK with rectangular envelope

Carrier Frequency: 0.21 (digital frequency)

Samples Per Bit: 16

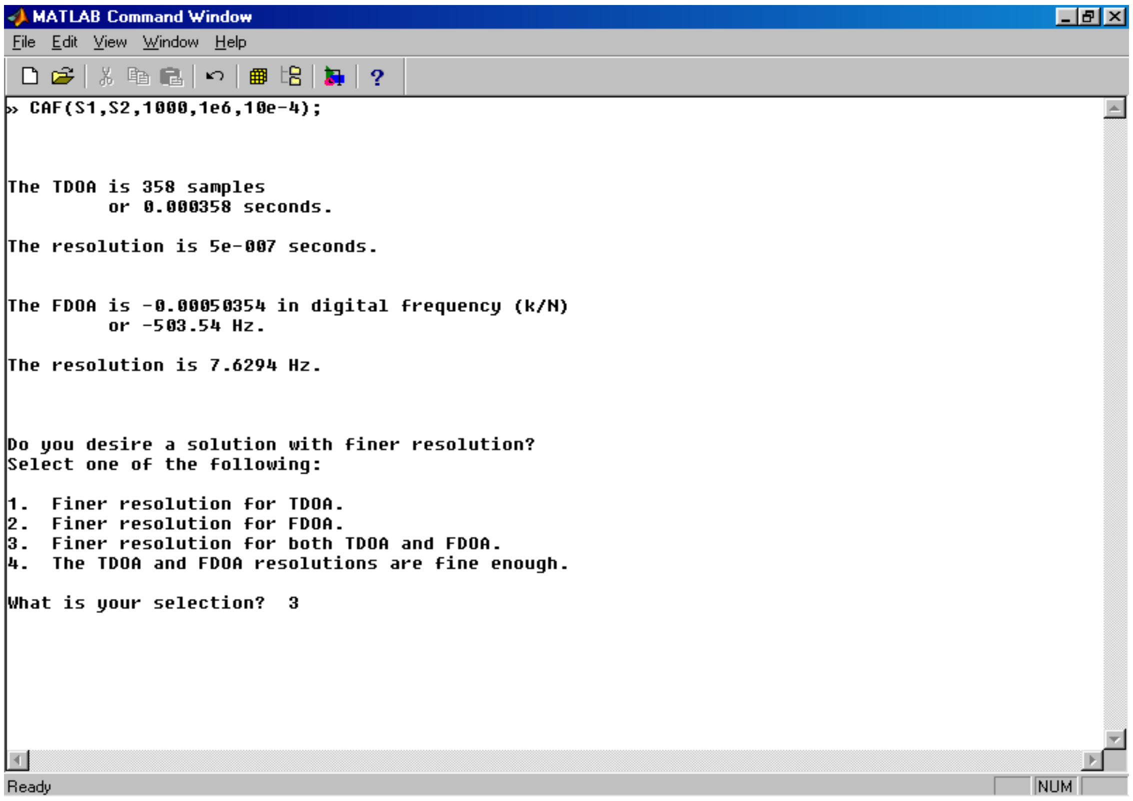
Signal-to-Noise Ratio for the two signals: 20 dB & 20 dB

Number of Samples: 65536

TDOA: 358 samples

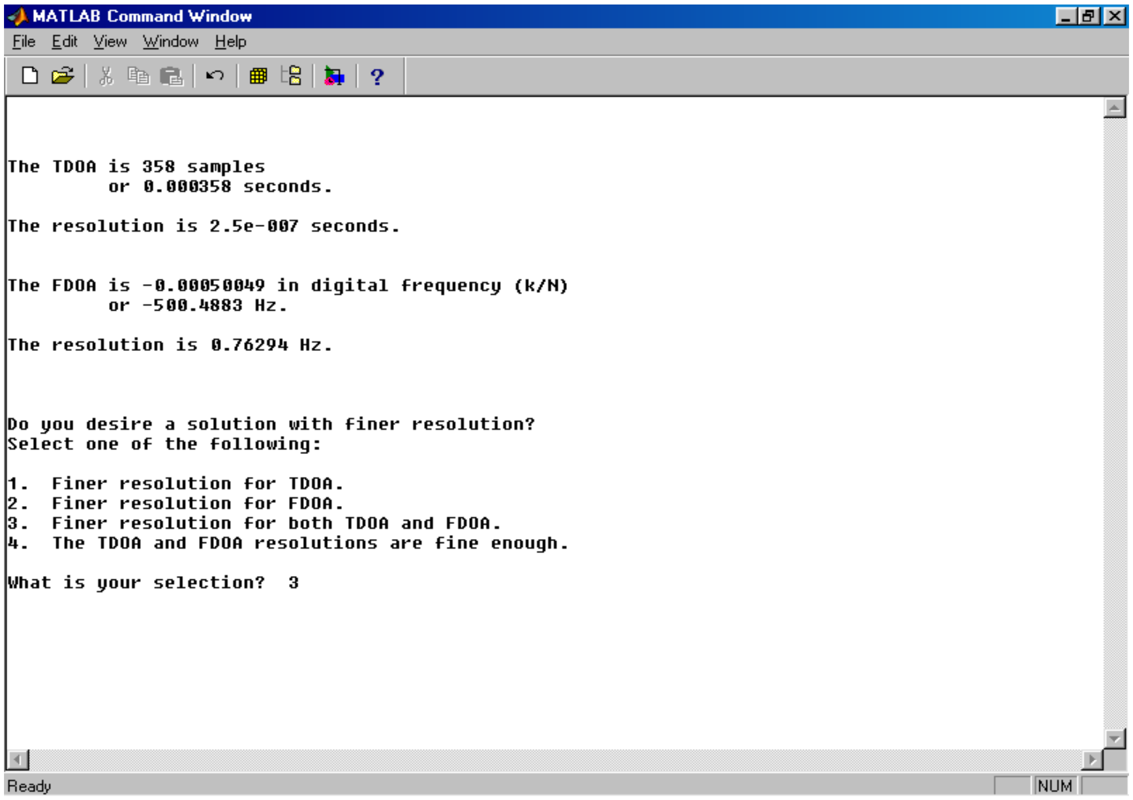
FDOA: – 0.0005 (digital frequency)

Figures (3-9) through (3-12) show the MATLAB command window results of running CAF.m on the signals with the above parameters. Again using the assumed value of f_s = 1 MHz, the expected TDOA is $3.58x10^{-4}$ seconds and the expected TDOA is –500 Hz. Note that the TDOA was computed exactly after the first iteration. After three more iterations, the FDOA was also computed exactly. Notice that with each subsequent iteration, the resolution of the TDOA calculation improves by a factor of two, and the FDOA resolution improves by a factor of 10, as expected. Figures (3-13) and (3-14) show the CAF surface in three dimensions and two dimensions, respectively. The plots are as expected, with the surface's peak occurring at a TDOA of exactly $3.58x10^{-4}$ seconds, and an FDOA that is within FFT method resolution of –500 Hz.



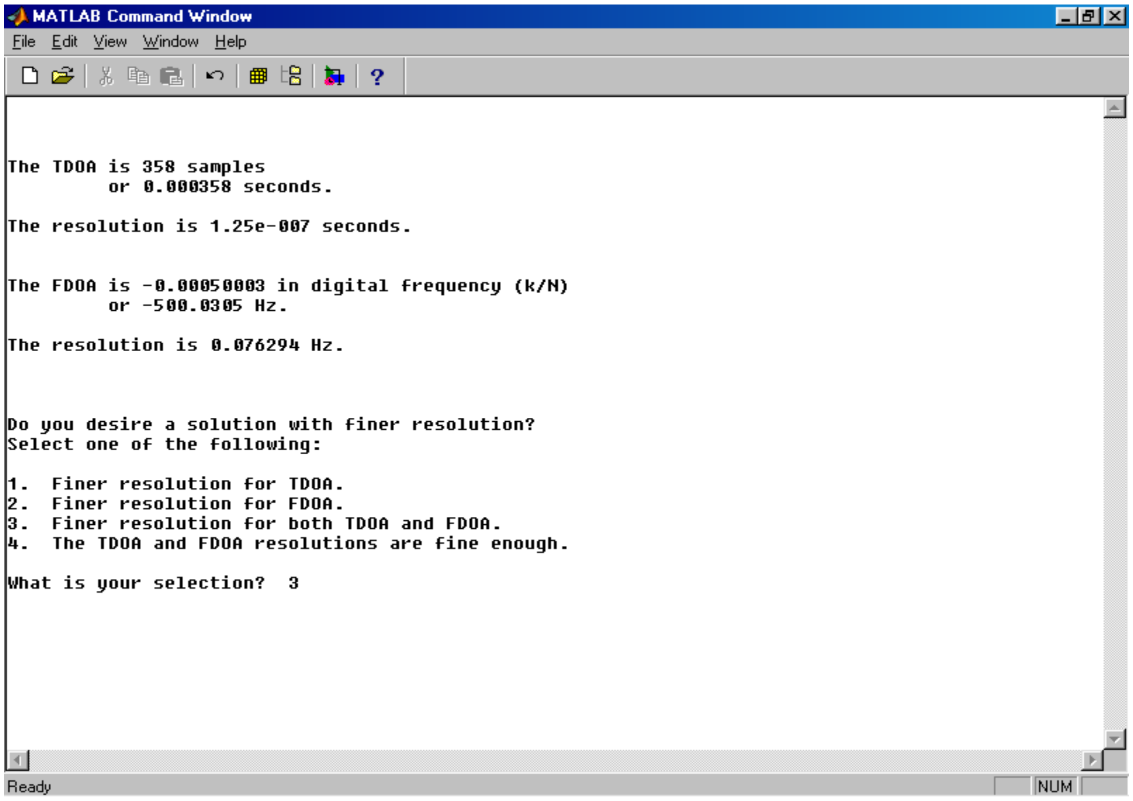
```
» CAF(S1,S2,1000,1e6,10e-4);


The TDOA is 358 samples
         or 0.000358 seconds.

The resolution is 5e-007 seconds.


The FDOA is -0.00050354 in digital frequency (k/N)
         or -503.54 Hz.

The resolution is 7.6294 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.   Finer resolution for TDOA.
2.   Finer resolution for FDOA.
3.   Finer resolution for both TDOA and FDOA.
4.   The TDOA and FDOA resolutions are fine enough.

What is your selection?  3
```

Figure 3-9. CAF.m Results – Case #3 (1st Iteration).



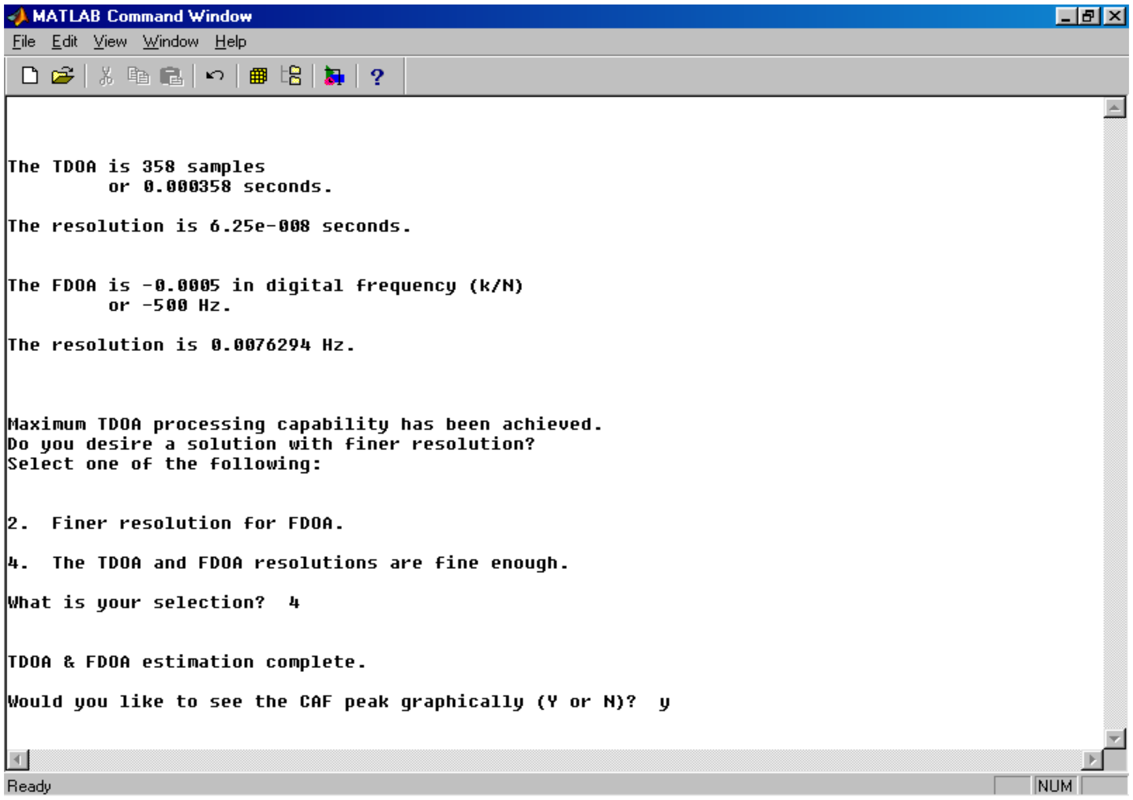
```


The TDOA is 358 samples
         or 0.000358 seconds.

The resolution is 2.5e-007 seconds.


The FDOA is -0.00050049 in digital frequency (k/N)
         or -500.4883 Hz.

The resolution is 0.76294 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.   Finer resolution for TDOA.
2.   Finer resolution for FDOA.
3.   Finer resolution for both TDOA and FDOA.
4.   The TDOA and FDOA resolutions are fine enough.

What is your selection?  3
```

Figure 3-10. CAF.m Results – Case #3 (2nd Iteration).



```
The TDOA is 358 samples
         or 0.000358 seconds.

The resolution is 1.25e-007 seconds.


The FDOA is -0.00050003 in digital frequency (k/N)
         or -500.0305 Hz.

The resolution is 0.076294 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  3
```

Figure 3-11. CAF.m Results – Case #3 (3rd Iteration).



```
The TDOA is 358 samples
         or 0.000358 seconds.

The resolution is 6.25e-008 seconds.


The FDOA is -0.0005 in digital frequency (k/N)
         or -500 Hz.

The resolution is 0.0076294 Hz.



Maximum TDOA processing capability has been achieved.
Do you desire a solution with finer resolution?
Select one of the following:


2.  Finer resolution for FDOA.

4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 3-12. CAF.m Results – Case #3 (4th Iteration).

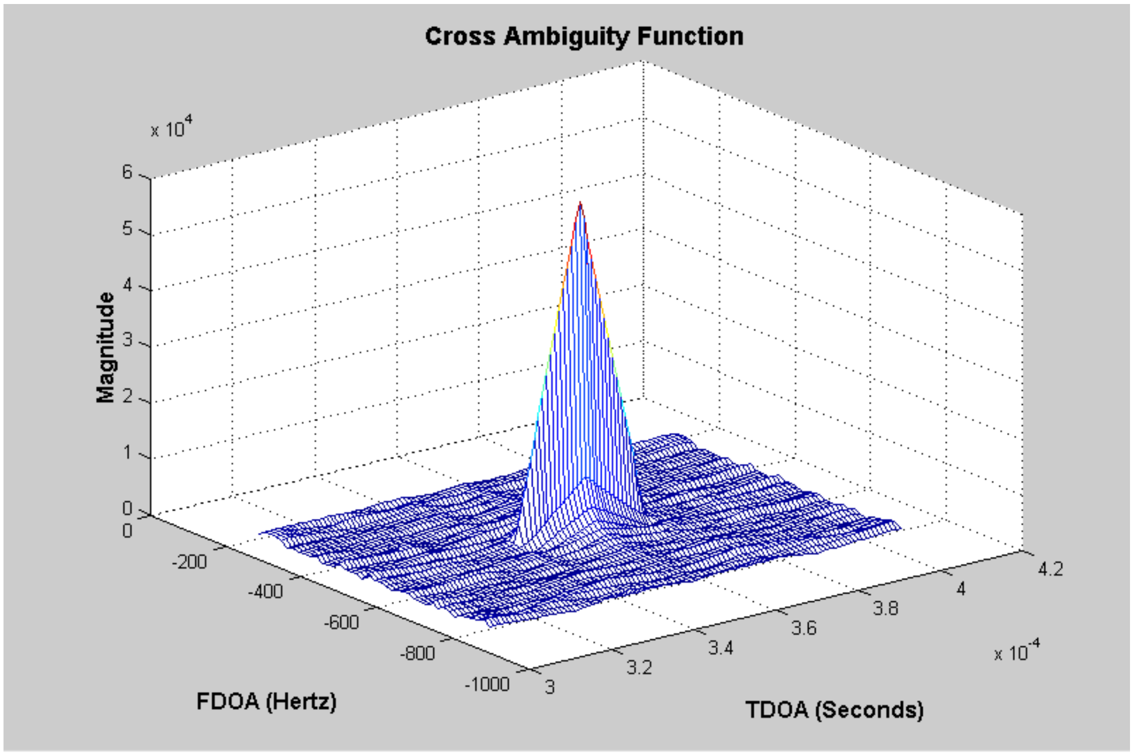


Figure 3-13. 3-D CAF Surface – Case #3.

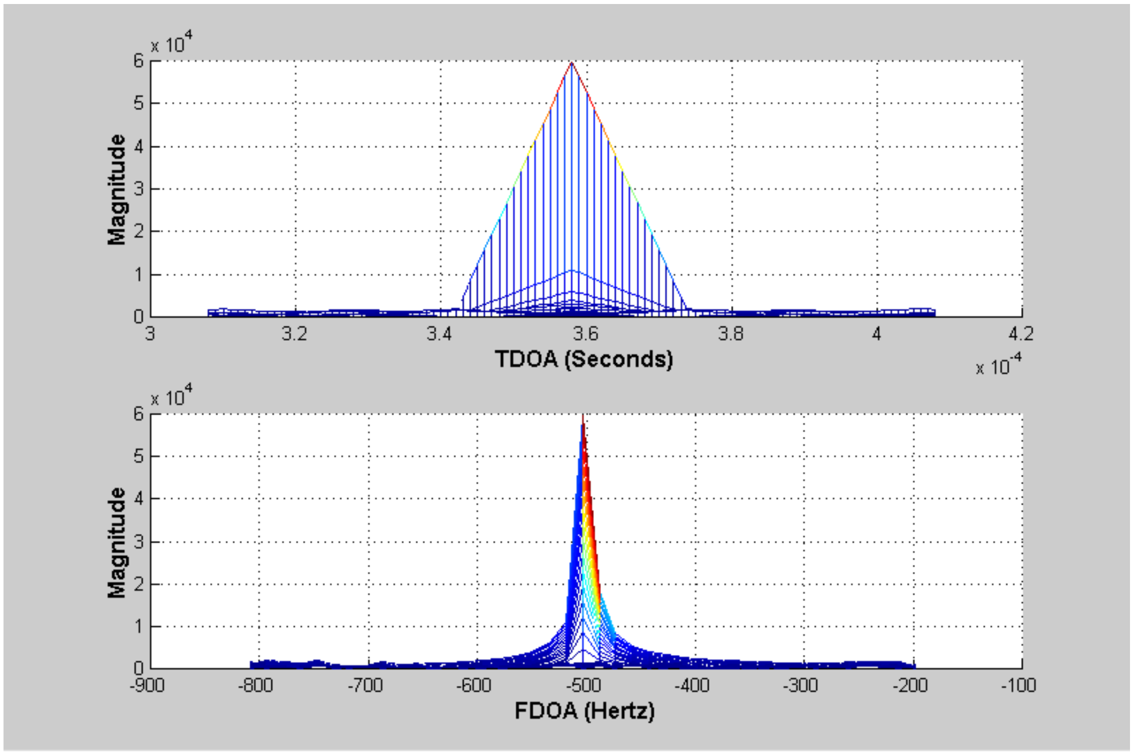


Figure 3-14. 2-D Cuts Through CAF Surface – Case #3.

For the final test, Case #4, signals with the following parameters were input into the CAF.m program:

Signal Type: BPSK with half-cosine envelope

Carrier Frequency: 0.21 (digital frequency)

Samples Per Bit: 16

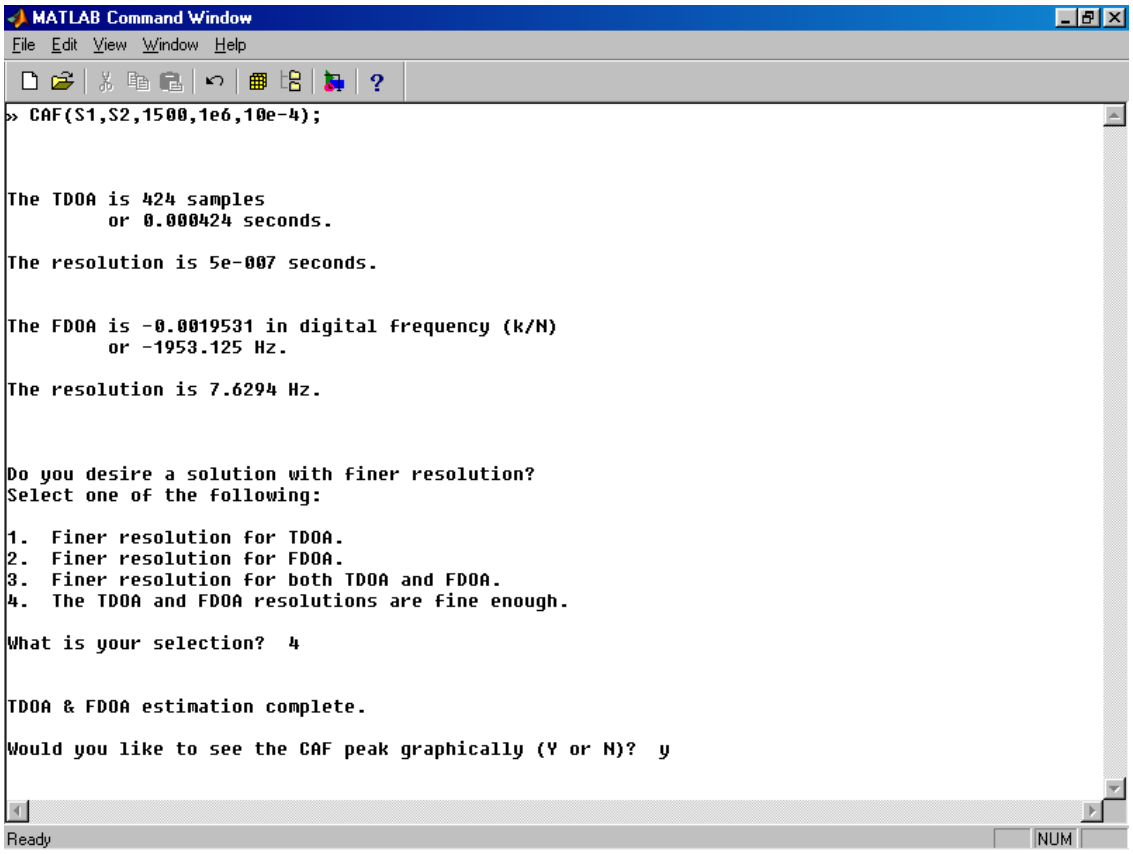
Signal-to-Noise Ratio for the two signals: 0 dB & -20 dB

Number of Samples: 65536

TDOA: 423 samples

FDOA: – 0.001953125   (digital frequency)

The expected TDOA is $4.23x10^{-4}$ seconds and the expected FDOA is –1953.125 Hz.



```
» CAF(S1,S2,1500,1e6,10e-4);



The TDOA is 424 samples
        or 0.000424 seconds.

The resolution is 5e-007 seconds.


The FDOA is -0.0019531 in digital frequency (k/N)
        or -1953.125 Hz.

The resolution is 7.6294 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 3-15. CAF.m Results – Case #4.

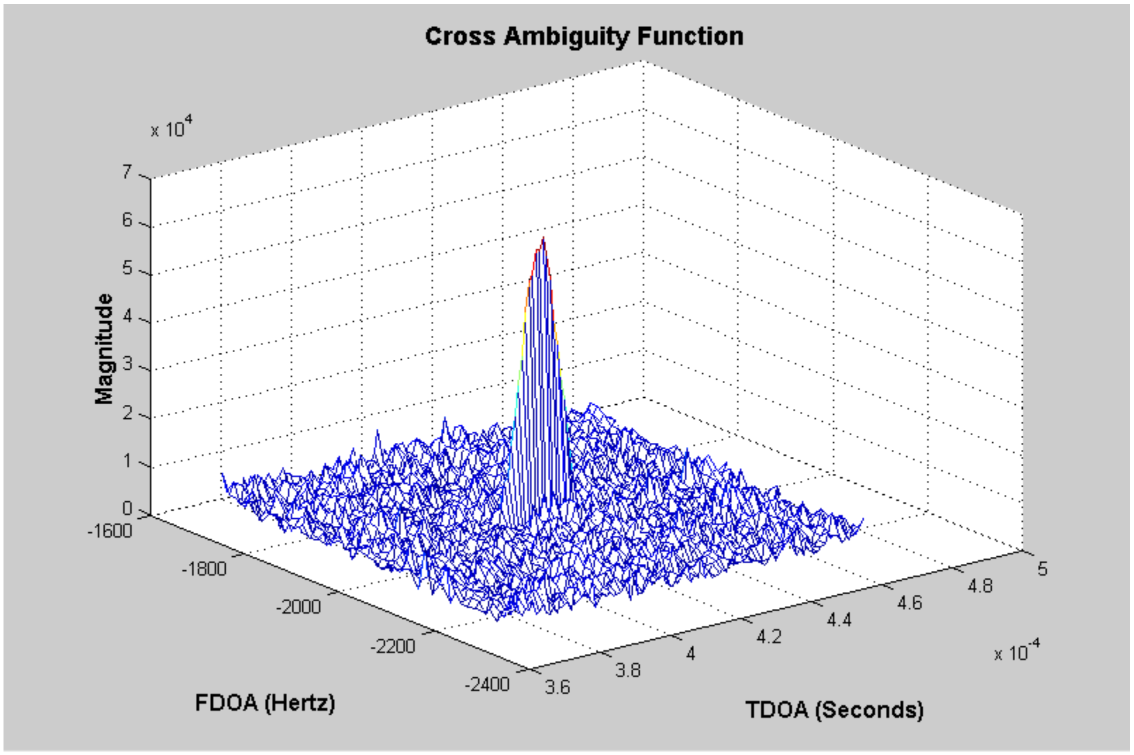


Figure 3-16. 3-D CAF Surface – Case #4.

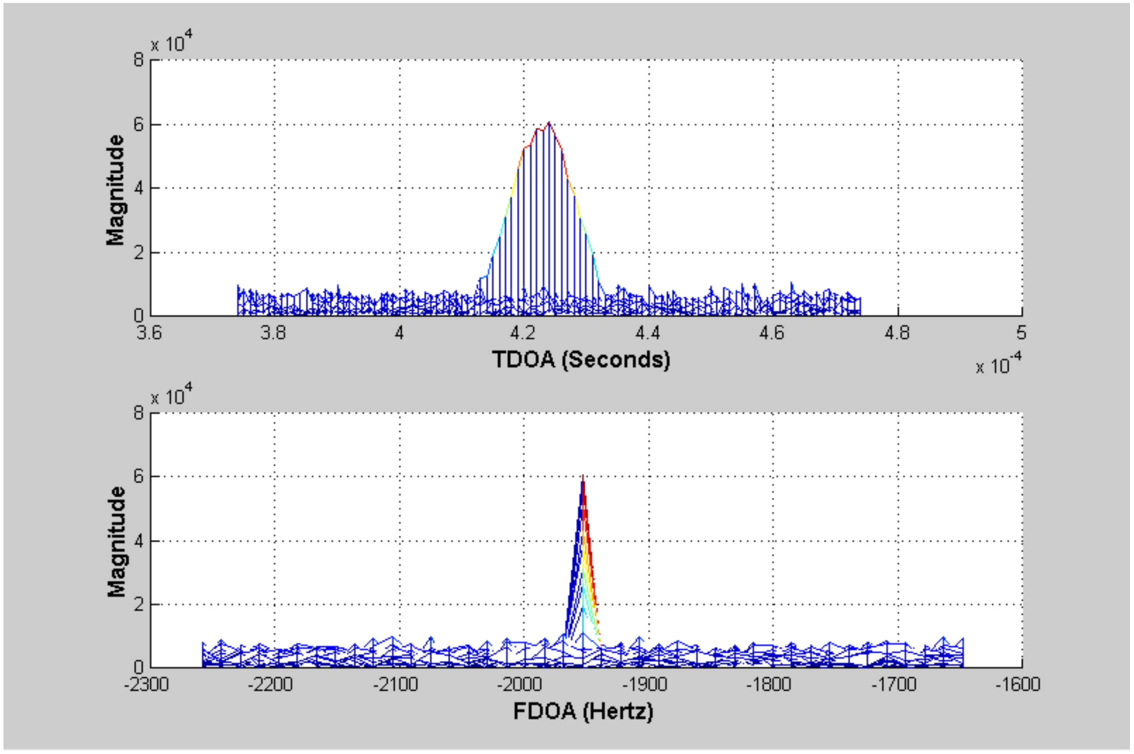


Figure 3-17. 2-D Cuts Through CAF Surface – Case #4.

Figure (3-15) shows the MATLAB command window results after running CAF.m, while Figures (3-16) and (3-17) show the 3-D and 2-D views of the surface, respectively. Note that the FDOA was computed exactly after the first iteration. This is because the digital frequency (–0.001953125) just happens to be associated with an integer frequency bin number, k. Since digital frequency is defined as $\frac{k}{N}$, and $N = 65536$, it follows that $k = 128$. Note also that the computed TDOA is $4.24x10^{-4}$ seconds, which is exactly one sample away from the actual TDOA of $4.23x10^{-4}$ seconds. Further iterations of the fine mode do not yield the exact TDOA. This is likely due to the noise in the signals, which can introduce errors. This issue will be discussed further in Chapter VI.

As the four different examples show, the CAF.m and CAF_peak.m programs function properly to compute accurate TDOAs and FDOAs and display the resulting CAF surfaces, respectively. As mentioned before, the signal sets used to validate the programs were not physically realizable due to the artificiality of the constant TDOAs and FDOAs. Chapter IV provides background information on BPSK signals, and also describes a Cartesian representation for practical emitter-collector geometries. Chapter V then describes the approach used in the sig_gen.m program, which generates BPSK signals based upon user-defined geometries. Several example signal sets are generated and then input into CAF.m. The results are analyzed and compared with theoretical TDOA and FDOA calculations.

IV. GEOMETRY-SPECIFIC SIGNAL GENERATION

A. BINARY-PHASE-SHIFT-KEYING SIGNALS

The MATLAB code in Appendix B is specifically designed to generate Binary-Phase-Shift-Keying (BPSK) signals. The BPSK technique is a very common digital modulation technique, and it is widely used in both military and commercial communications systems [6].

In BPSK modulation, a sinusoidal carrier wave is modulated by a data signal consisting of the binary digits "1" and "0." The data signal shifts the phase of the carrier waveform to one of two states, either zero or 180° (or π radians). Due to the familiar trigonometric relationships

$$\begin{aligned} \sin(x+\pi) &= -\sin(x) \\ \cos(x+\pi) &= -\cos(x), \end{aligned} \tag{4-1}$$

it is clear that the two possible states in a BPSK system are simply the carrier multiplied by ±1.

The general analytic expression for a BPSK signal is:

$$s_i(t) = A\cos[2\pi f_0 t + \varphi_i(t)] \qquad \begin{cases} 0 \le t \le T_{sym} \\ i = 1, 2 \end{cases} \tag{4-2}$$

where A is simply the amplitude of the carrier, f_0 is the carrier frequency, $\varphi_i(t)$ takes on the values of zero or π, and T_{sym} is the data symbol period. In binary modulation techniques, a symbol consists of just one data bit, either 0 or 1. Therefore, in binary systems such as BPSK, the terms "data symbol" and "symbol period" are synonymous with "data bit" and "bit period," respectively. Using Equation (4-2) and bearing Equations (4-1) in mind, the two possible waveforms transmitted in a BPSK signal are:

$$\begin{aligned} s_1(t) &= A\cos(2\pi f_0 t) \\ s_2(t) &= -A\cos(2\pi f_0 t) \end{aligned} \tag{4-3}$$

Equations (4-3) represent continuous-time or analog signals. In the discrete-time or sampled case, the following representations are used:

$$\begin{aligned} s_1(n) &= A\cos[2\pi f_0(nT_s)] \\ s_2(n) &= -A\cos[2\pi f_0(nT_s)] \end{aligned} \tag{4-4}$$

The convention used throughout this thesis is that a data bit 0 is represented by $s_1(n)$ and a data bit 1 is transmitted as $s_2(n)$.

Figure (4-1) shows an example of a sampled BPSK signal modulated with the data stream [0 1 0 1]. The signal was sampled at f_s = 10 kHz and has the following parameters: f_0 = 650 Hz and the symbol rate $R_{sym} = \frac{1}{T_{sym}}$ = 200 bits per second (bps).

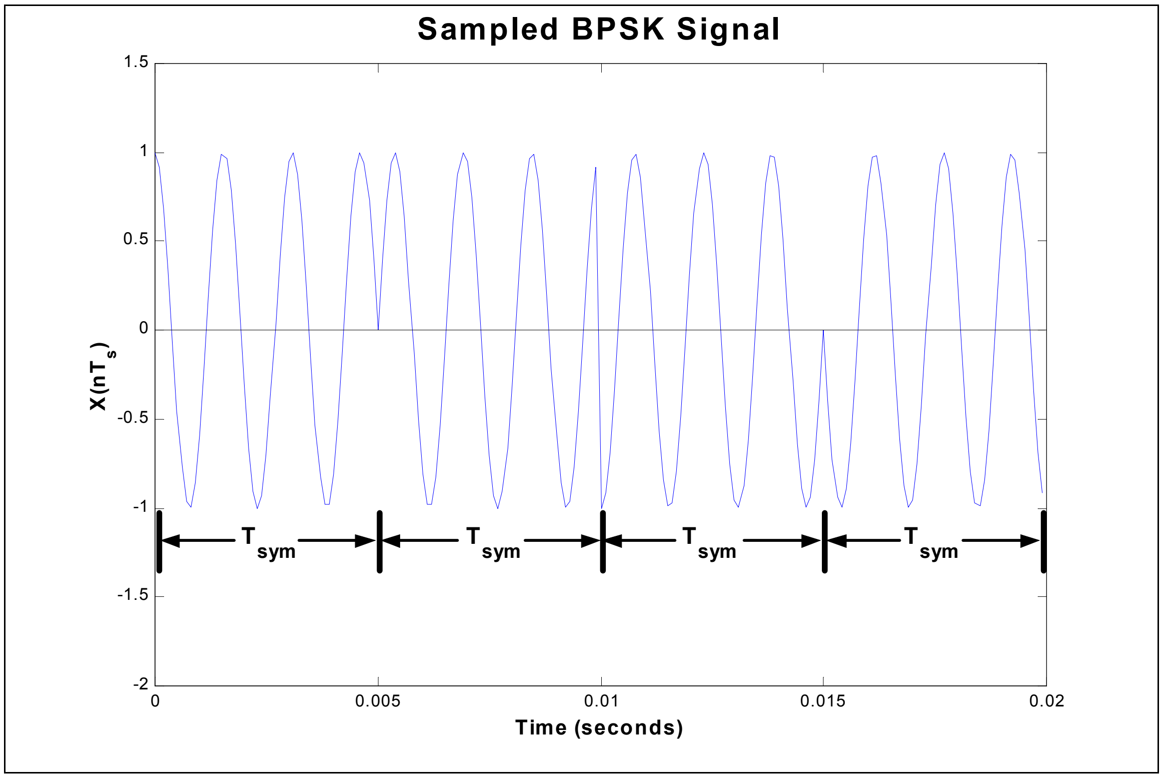


Figure 4-1. Example of a BPSK Signal (After [6]).

Notice that the beginning of the transmitted signal is a cosine wave, represented as $s_1(n)$ and indicating that the first data bit is a 0 in accordance with Equations (4-4). As expected, $s_1(n)$ continues until the end of the symbol interval is encountered at 0.005 s, which is the reciprocal of the defined R_{sym}. At this point, the next bit in the data stream is transmitted. Since the phase has changed by π radians, $s_2(n)$ is the transmitted signal and the data bit must therefore be a 1. Had the next data bit been a 0 instead, $s_1(n)$ would have been transmitted for another period and no phase change would have been detected in the signal. By looking at each symbol period, it is easy to determine that the transmitted data stream is indeed [0 1 0 1].

B. EMITTER – COLLECTOR GEOMETRY

Real-world collection systems often employ a pair of separate collectors. The signals received by the individual collectors are from the same transmitter, but shifts in time and frequency are inherent due to the different paths traveled by the two signals. In these configurations, the two received signals can be processed to determine the TDOA and FDOA between the two collectors. With exact knowledge of the collectors' positions, successive TDOA and FDOA measurements can be plotted to determine the location of the associated emitter.

No known signal generation software gives the ability to create signals that are based upon specific emitter-collector geometries. Some programs can generate signal pairs that have constant TDOAs and FDOAs embedded in them, but this does not accurately model real-world systems whose geometries change with time due to non-zero relative velocities between emitters and collectors. Figure (4-2) shows a simple model of a generic emitter-collector geometry. Cartesian coordinates simplify the model by assuming that the emitter and collectors are moving on or around a flat earth. Obviously, real-world systems are three-dimensional, but Figure (4-2) is in two dimensions solely for ease of depiction. Imagine that a Z-axis is perpendicular to the paper.

In Figure (4-2), C_1, C_2, and E represent the two collectors and the emitter, all located at the coordinate positions shown. The symbols $\mathbf{r_1}$ and $\mathbf{r_2}$ are the relative position vectors between each of the collectors and the emitter, while $\mathbf{v_{C1}}$, $\mathbf{v_{C2}}$, and $\mathbf{v_E}$ are the respective velocity vectors. It is important to note that Figure (4-2) represents an instantaneous "snapshot" of a generic system's geometry. Since the emitter and/or collectors are moving at their respective velocities, the geometry changes with each passing instant of time. This is precisely why the TDOAs and FDOAs in a system are time-varying in nature. Chapter V will describe in detail the software in Appendix B, which creates geometry-specific signals that capture the time-varying quality of the TDOA and FDOA. Chapter V will also show that the software in Appendix B works properly by showing the results of inputting generated signals into the CAF software in Appendix A.

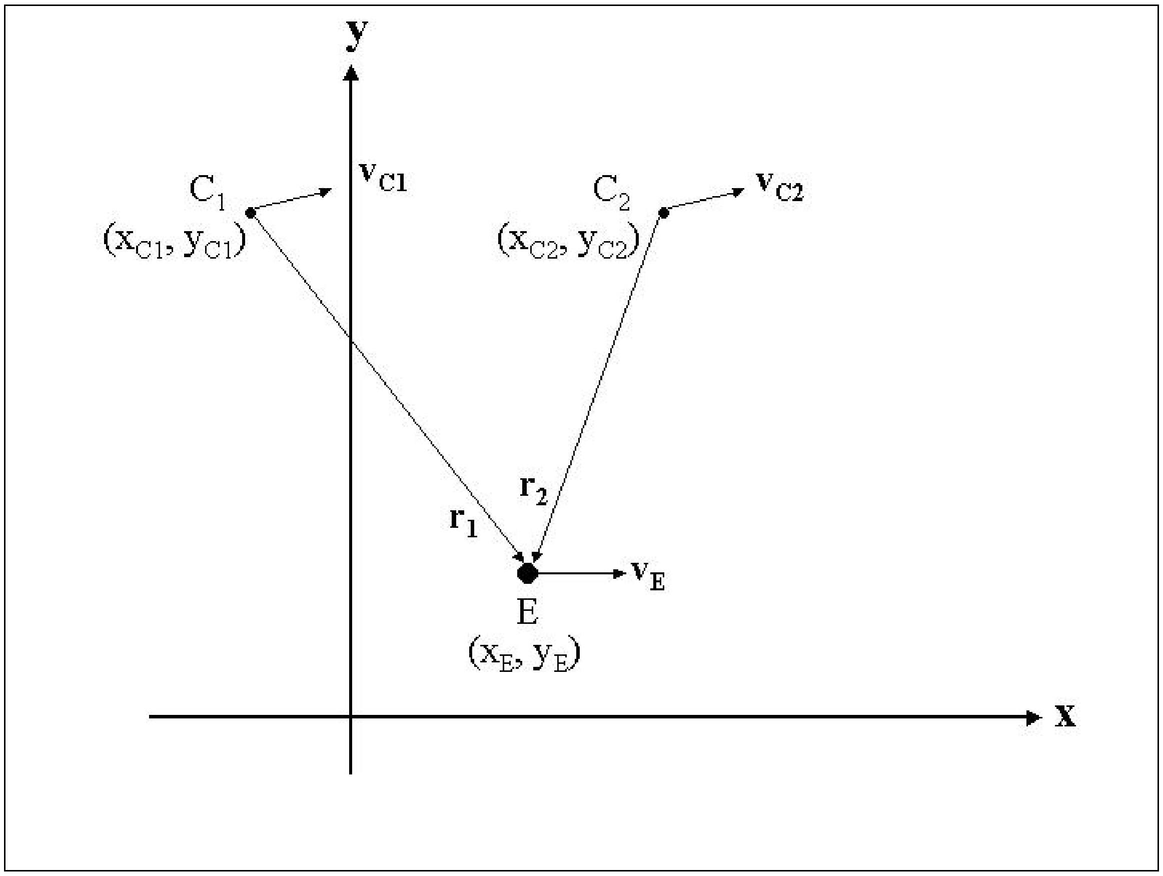


Figure 4-2. 2-D Emitter-Collector Geometry (After [7]).

C. CALCULATING THEORETICAL TDOA(S) AND FDOA(S)

The TDOA between two signals is simply the difference in time that it takes two signals to travel down their respective paths from the emitter to the associated receivers. For the geometry shown in Figure (4-2), the TDOA between C_1 and C_2 is therefore [7]:

$$TDOA = \frac{|\mathbf{r}_2| - |\mathbf{r}_1|}{c}, \tag{4-5}$$

where $|\mathbf{r_2}|-|\mathbf{r_1}|$ is the difference in length between the two paths and c is the speed of light, at which the signals travel. The vectors $\mathbf{r_2}$ and $\mathbf{r_1}$ are determined simply by calculating the difference between their x and y coordinates and those of the emitter, so that

$$\begin{aligned}\mathbf{r_1} &= \begin{bmatrix} x_E - x_{C1} \\ y_E - y_{C1} \end{bmatrix} \\ \mathbf{r_2} &= \begin{bmatrix} x_E - x_{C2} \\ y_E - y_{C2} \end{bmatrix}\end{aligned} \tag{4-6}$$

Now, using the Pythagorean Theorem to define the magnitudes of the path vectors $\mathbf{r_1}$ and $\mathbf{r_2}$, Equation (4-5) becomes

$$TDOA = \frac{1}{c}\left[\sqrt{(x_E - x_{C2})^2 + (y_E - y_{C2})^2} - \sqrt{(x_E - x_{C1})^2 + (y_E - y_{C1})^2}\right] \tag{4-7}$$

Of course, Equation (4-7) is strictly for a two-dimensional geometry, but it can easily be expanded for the 3-D case as well:

$$\begin{aligned}TDOA = \frac{1}{c}\Big[&\sqrt{(x_E - x_{C2})^2 + (y_E - y_{C2})^2 + (z_E - z_{C2})^2} \\ &-\sqrt{(x_E - x_{C1})^2 + (y_E - y_{C1})^2 + (z_E - z_{C1})^2}\Big]\end{aligned} \tag{4-8}$$

Equation (4-8) is used to calculate the theoretical TDOAs for generated signal pairs. In Chapter V, theoretical values are compared to the results produced by the CAF software.

The FDOA between two signals is simply the difference between their two Doppler shifts. From Figure (4-2), the Doppler shift between one of the collectors and the emitter can be defined as

$$\delta f = \frac{f_0}{c} v, \tag{4-9}$$

where f_0 is the signal's *constant* carrier frequency, c is the speed of light, and v is the (scalar) velocity of closure between the collector and emitter. The velocity of closure is simply defined as the component of the relative velocity that is along the path of propagation. Since the collector and emitter velocities and the associated propagation path are vector quantities, the velocity of closure (v in Equation (4-9)) must be calculated with the vector dot product as follows [8]:

$$v = \frac{\mathbf{v} \cdot \mathbf{r}}{|\mathbf{r}|}, \tag{4-10}$$

where **r** represents the path vector and **v** is the relative velocity between the collector and emitter, as follows:

$$\mathbf{v} = \begin{bmatrix} v_{Ex} - v_{Cx} \\ v_{Ey} - v_{Cy} \end{bmatrix} \tag{4-11}$$

In Equation (4-11), v_{Ex} and v_{Cx} represent the velocities of the emitter and collector in the x-direction, while v_{Ey} and v_{Cy} are the y components of the velocities. Now, substituting Equation (4-10) into Equation (4-9), the Doppler shift for a collector and emitter pair is:

$$\delta f = \frac{f_0}{c}\left(\frac{\mathbf{v} \cdot \mathbf{r}}{|\mathbf{r}|}\right), \tag{4-12}$$

Substituting Equations (4-11) and (4-6) into Equation (4-12) and simplifying produces the form of the Doppler shift in two dimensions:

$$\delta f = \frac{f_0}{c}\left[\frac{(v_{Ex} - v_{Cx})(x_E - x_C) + (v_{Ey} - v_{Cy})(y_E - y_C)}{\sqrt{(x_E - x_C)^2 + (y_E - y_C)^2}}\right] \tag{4-13}$$

Now, since the FDOA is defined as the difference between the two Doppler shifts associated with each of the collectors and the emitter, FDOA can be expressed as:

$$\begin{aligned} FDOA &= \delta f_2 - \delta f_1 \\ &= \frac{f_0}{c}\left[\frac{(v_{Ex} - v_{C2x})(x_E - x_{C2}) + (v_{Ey} - v_{C2y})(y_E - y_{C2})}{\sqrt{(x_E - x_{C2})^2 + (y_E - y_{C2})^2}}\right. \\ &\qquad \left. - \frac{(v_{Ex} - v_{C1x})(x_E - x_{C1}) + (v_{Ey} - v_{C1y})(y_E - y_{C1})}{\sqrt{(x_E - x_{C1})^2 + (y_E - y_{C1})^2}}\right] \end{aligned} \tag{4-14}$$

Equation (4-14) is the FDOA for a two dimensional geometry such as that found in Figure (4-2). It is a straightforward process to extend Equation (4-14) to a three dimensional geometry:

$$FDOA =$$

$$\frac{f_0}{c}\left[\frac{(v_{Ex}-v_{C2x})(x_E-x_{C2})+(v_{Ey}-v_{C2y})(y_E-y_{C2})+(v_{Ez}-v_{C2z})(z_E-z_{C2})}{\sqrt{(x_E-x_{C2})^2+(y_E-y_{C2})^2+(z_E-z_{C2})^2}} - \frac{(v_{Ex}-v_{C1x})(x_E-x_{C1})+(v_{Ey}-v_{C1y})(y_E-y_{C1})+(v_{Ez}-v_{C1z})(z_E-z_{C1})}{\sqrt{(x_E-x_{C1})^2+(y_E-y_{C1})^2+(z_E-z_{C1})^2}}\right] \quad (4\text{-}15)$$

Equation (4-15) is used to calculate the theoretical FDOAs for generated signal pairs. In Chapter V, theoretical values are compared to the results produced by the CAF software.

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V. IMPLEMENTING THE SIGNAL GENERATOR

A. ANALYSIS OF THE SIGNAL GENERATION SOFTWARE

1. The "Sig_gen.m" Program

The program sig_gen.m, listed in Appendix B, is a MATLAB function that generates pairs of BPSK signals according to user-defined signal parameters and collector-emitter geometries (of the form described in section IV.B). The function is invoked in the MATLAB command window with a line of the form:

[Sa1, Sa2, S1, S2] = sig_gen;

There are no input arguments to the function, since the user is queried for all required parameters. The four output arguments are returned as signal vectors. *Sa1* and *Sa2* are the two generated signals in *analytic signal* format, which is required for subsequent CAF computations. *S1* and *S2* are the real-valued, time domain representations of the same two signals. The real signals are made available in case the user desires to plot the signals in the time domain, or to look at the periodogram of the real data, etc.

The first section of the function queries the user for all information required to generate the signals. The user is first asked to input the position and velocity vectors of the two collectors and the emitter at "time 0." All position and velocity vectors are entered in the form "[X Y Z]," where X, Y, and Z denote the components of position and velocity in each of the 3-D Cartesian directions. Position elements are expressed in meters and velocity elements are expressed in meters per second. All velocities are assumed to be constant during the period of collection. Note that "time 0" represents the beginning of the collection period *onboard the collectors*. This is an important point that will be discussed later in this section. The two collectors are assumed to have exactly synchronized time clocks. With the geometry inputs completed, the user is asked to define the following signal parameters: carrier frequency (*f0*) in Hz, sampling frequency (*fs*) in Hz, the total number of samples to generate (*N*), the ratio of symbol energy to noise power spectral density at each collector (*Es_No1* and *Es_No2*) in dB, and the symbol rate (*Rsym*) in symbols per second. The function calculates the sample period directly from the sampling frequency as $Ts = \frac{1}{fs}$. Likewise, the symbol period is calculated as $Tsym = \frac{1}{Rsym}$. Finally, *Es_No1* and *Es_No2* are converted from dB to ratio values for future computations. The relationship used is:

$$\frac{E_s}{N_0}(dB) = 10\log_{10}\frac{E_s}{N_0} \quad \text{and} \quad \frac{E_s}{N_0} = 10^{\frac{1}{10}\frac{E_s}{N_0}(dB)} \tag{5-1}$$

Next, the speed of light constant (*c*) is defined as $2.997925\text{x}10^8\, m/s$. The amplitude (*A*) for the signals is defined to be equal to one. The choice of this value is simply for convenience; any value could be used here. The average signal power (*Ps*) is then calculated, since it will be needed to compute the noise power necessary to achieve the user's desired $\frac{E_s}{N_0}$. The definition of a periodic signal's average power is [6]:

$$P_s = \frac{1}{T_0}\int_0^{T_0} s^2(t)dt \tag{5-2}$$

Using either $s_1(t)$ or $s_2(t)$ from Equations (4-3) and substituting into Equation (5-2),

$$P_s = \frac{1}{T_0}\int_0^{T_0} A^2\cos^2(2\pi f_0 t)dt \tag{5-3}$$

Simplifying Equation (5-3) leads to the result:

$$P_s = \frac{A^2}{2} \tag{5-4}$$

Sig_gen.m uses Equation (5-4) to calculate the constant *Ps* directly.

Next, the program creates the noise components of the two signals, according to the $\frac{E_s}{N_0}$ defined by the user. The noise modeled by the program is Additive White Gaussian Noise (AWGN), which has a mean of zero and a variance that can be determined as follows. From [6], E_s represents the amount of energy in one symbol, and can be described as the average signal power times the symbol period:

$$E_s = P_s T_{sym} \tag{5-5}$$

The noise power spectral density, N_0, can be described as the noise power divided by the bandwidth:

$$N_0 = \frac{P_n}{B} \tag{5-6}$$

Dividing Equation (5-5) by Equation (5-6),

$$\frac{E_s}{N_0} = \frac{P_s T_{sym}}{P_n / B} = \frac{P_s T_{sym} B}{P_n} \tag{5-7}$$

Rearranging Equation (5-7)to determine the noise power,

$$P_n = \frac{P_s T_{sym} B}{E_s / N_0} \tag{5-8}$$

From [11], P_n for AWGN is simply equal to its variance, σ^2. Since the signals are generated digitally, the noise power is spread throughout the total range of digital frequencies from $-\frac{1}{2}$ to $\frac{1}{2}$. This makes the bandwidth, B, equal to one. This leads to:

$$\sigma^2 = \frac{P_s T_{sym} B}{E_s / N_0} \quad (where\, B = 1) \tag{5-9}$$

The built-in MATLAB function *randn* produces random values from a Gaussian distribution with a mean of zero and a variance of one. In order to generate noise with the proper power, the *randn*-generated numbers must be properly scaled. Using the following property [11]:

$$\text{var}(c \cdot x) = c^2\, \text{var}(x), \tag{5-10}$$

it is clear that the scaling factor is σ (not σ^2). Therefore, sig_gen.m calculates σ by taking the square root of both sides of Equation (5-9). Finally, the two AWGN vectors (one for each collector) are generated by multiplying the unit-variance values produced by *randn* by the respective σ values. Each AWGN vector contains N values of noise, which will eventually be added, element-by-element, to the N computed values of the repective signals. The noise vectors are called *Noise1* and *Noise2*.

Next, the program computes a position matrix for each of the two collectors. This is a straightforward process since the user provides the starting position and the velocity for the collectors. The position of a collector at each sample period is simply equal to its position at the last sample plus a distance equal to the velocity times the sample period. Using two for loops, sig_gen.m calculates the position of each collector at each sample time, from 1 to N. The resulting position matrices are N x 3, since a 1 x 3 vector (of the form [X Y Z]) defines a collector's position at each of the N sample times.

The next major portion of sig_gen.m computes the position matrix and time vectors for the emitter. Again, note that time 0 refers to the time onboard the collectors when the signal collection begins (i.e., nT_s, where $n = 0$). Successive times onboard the collectors are simply multiples of the sampling period. Since the signal must travel from the emitter along two separate, non-zero paths to the collectors, it takes different amounts of time for the signal to travel to the two receivers. Therefore, for samples taken at the receivers at time nT_s, the emitter will have transmitted those portions of the signal at two different times that are less than nT_s by amounts equal to the time it takes the signals to travel down the respective paths. For example, assume that two collectors are positioned such that one is 1000 km from the emitter, and the other is 1100 km from the emitter. The sample taken at the first collector at time 0 will represent the portion of the signal transmitted at time ($0 - \dfrac{1x10^6 m}{2.997925x10^8 m/s}$), or –0.003336 seconds. On the other hand, the sample taken at the second collector at time 0 will have been transmitted at time ($0 - \dfrac{1.1x10^6 m}{2.997925x10^8 m/s}$), or –0.003669 seconds. In order to capture this timing arrangement, the transmit times corresponding to each sample must be tracked for each collector. Since the emitter may be moving, its position must also be tracked. In order to compute its position, the transmit times for each sample must be used along with the user-defined velocity and starting position. Since the position of the emitter is needed to figure out the distance of the path between it and the collectors, it is clear that the time and position of the emitter are interdependent. They must therefore be determined in an iterative fashion as described in the next paragraph.

A while loop is used to determine the time and position of the emitter for the first sample (i.e., the one that arrives at the collector at time 0) at collector number one. The initial estimates are that the transmit time is 0 and the position is the one entered by the user. Within the loop, the time is "updated" to be zero minus the path distance divided by the speed of light. The path distance is that between the collector's position and the current estimate of the emitter's position. The position of the emitter is then computed as the initial position plus the revised time multiplied by the emitter's velocity. Note that the time here is negative, so that the emitter is successively being "walked back" to where it was when it emitted the first sample. This iterative process continues in the while loop until two successive estimations of the time differ by less than one period of the carrier (i.e., $\frac{1}{f_0}$). When the loop ends, the computed time and position of the emitter are stored in a time vector (*t1*) and position matrix (*Pe1*), respectively. The process just described is then repeated in order to determine the time and position of the emitter corresponding to the first sample at the second collector. The emitter's time vector and position matrix associated with collector two are *t2* and *Pe2*, respectively.

Next, the "beginning" of the collected signal's data stream must be determined. The program compares the emitter times corresponding to the first sample taken at the two collectors. The earlier of the two times defines the starting point (called *StartPoint* in the code) of the data stream. The data symbols are stored as a vector (*P*) of zeros and ones, and the correct index into that vector must be computed for each of the two collectors in order to ensure that bit changes occur at the right times. In the program, the two indexes are called *SymbolIndex1* and *SymbolIndex2*. Since the earliest transmit time at the emitter is defined to be the starting point of the data stream, the index into the data vector is equal to one for the associated collector. The other collector, then, will have an index into the data vector that is equal to one plus the difference between the two transmit times divided by the symbol period. For example, assume that transmit times are –1 second for collector number one and –3 seconds for collector number two, and that the symbol period is one second. Collector two has the earlier transmit time, and therefore

SymbolIndex2 is equal to one. The transmit time for collector one is two seconds later, which corresponds to two symbol periods. *SymbolIndex1* must therefore be equal to three, or two greater than collector two's index.

Now that the emitter's position and transmit time for each collector's first sample have been computed, they must be determined for the remaining samples (i.e, numbers 2 through *N*). This is accomplished with a for loop running from 2 to *N*. Nested inside is a while loop very similar to the ones used to compute the transmit times and positions for the first sample (described above). The main difference is that for each sample, the initial estimate of the emitter's position is equal to the position for the previous sample plus the sample period times the emitter's velocity. Then, inside the while loop, the transmit time is computed as the sample time (nT_s) minus the path distance divided by the speed of light. The path distance here is that between the collector's position at time nT_s and the current estimate of the emitter's position. The position of the emitter is then computed as the initial position plus the elapsed time (i.e., the difference between the transmit time for the first sample and the estimate of the current sample's transmit time) multiplied by the emitter's velocity. The while loop ends when the difference between two successive time estimates is less than one period of the carrier. When the overall for loop is done, the *t1* vector contains *N* elements, each of which represents the time at which the associated sample was actually transmitted by the emitter. Likewise, the *Pe1* matrix contains *N* (1 x 3) vectors, each representing the emitter's position at the corresponding transmit times in *t1*. The process just described is then repeated for collector two, so that all the values for the *t2* vector and *Pe2* matrix are calculated.

Next, the data sequence is randomly generated and stored in the vector *P*. The possible values are 0 and 1, and they are assumed to be equally likely to occur. Therefore, MATLAB's *rand* function is used to create a vector of random numbers between 0 and 1. All numbers that are less than 0.5 are called 1s, and all numbers greater than 0.5 are called 0s. Note that prior to creating the random data bits, the program initializes the *rand* function to a specific seed. This is useful for testing purposes because it ensures that the same random data stream is generated every time the program is run. The line "*rand('seed',5);*" can simply be removed if a different random data stream is desired each time. Or, greater control could be given to the user by making the seed number an input to the program. Alternatively, if a user desired to have a specific data stream, the entire vector *P* could be made an input argument.

Finally, the actual transmitted signals are generated, sample-by-sample. Equation (4-2) is computed directly for each collector's signal, with *t* equal to the transmit times computed and stored in the vectors *t1* and *t2*. The phase component, $\varphi_i(t)$, is equal to the associated data bit stored in *P* (indexed by *SymbolIndex1* and *SymbolIndex2*) multiplied by π (and hence giving values equal to either 0 or π, as required). The noise elements from *Noise1* and *Noise2* are also added to their respective signal components. For each of the two signals, a for loop is used to compute all *N* samples. For each sample's computation, a test is made to determine if the total elapsed time has crossed into the next symbol period. If so, the appropriate index is incremented by one, so that the next data bit is obtained from *P*. If the next symbol period has not yet been reached, then the current data bit remains in effect. At the end of the two for loops, the vectors *S1* and *S2* contain the real-valued sampled signals that are received by collectors one and two, respectively. Using MATLAB's *hilbert* function, *Sa1* and *Sa2* are computed. They represent the complex-valued *analytic signal* representations of *S1* and *S2*, respectively. Note that although the Hilbert Transform is defined as in Equation (2-4), the *hilbert* function in MATLAB computes the analytic signal, Equation (2-3), directly. The four resulting signal vectors are returned to the user as output arguments.

After the signals have been created, sig_gen.m calls the tdoa_fdoa.m function, which computes and displays the theoretical values of the TDOA and FDOA at the beginning and end of the collection period. The toda_fdoa.m program is briefly described in the next section.

2. The "Tdoa_fdoa.m" Program

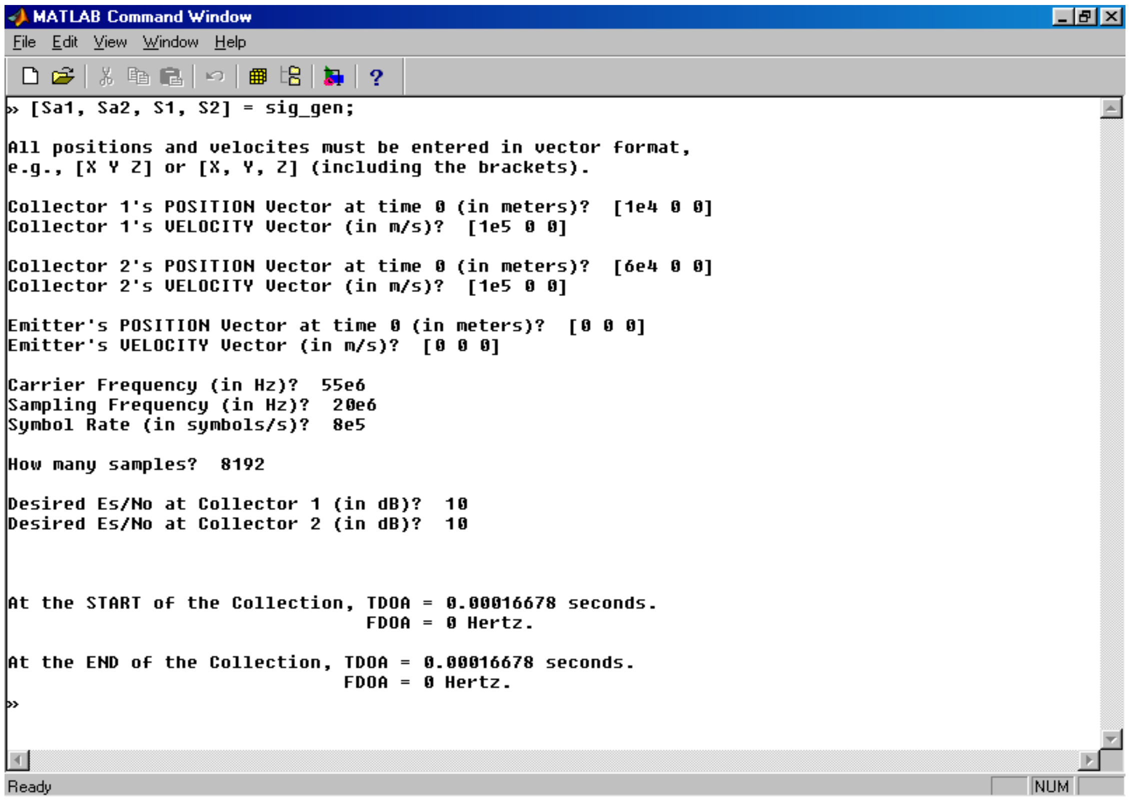
The tdoa_fdoa.m program takes a number of input arguments and computes the expected TDOA and FDOA at both the beginning and end of a signal collection. Again, real-world geometries produce time-varying TDOAs and FDOAs, so it is convenient to know what the expected range of values for each will be. The function's input arguments are: the carrier frequency (*f0*), the emitter's beginning and ending position vectors relative to each signal (*Pe1_b*, *Pe1_e*, *Pe2_b*, and *Pe2_e*), the beginning and ending position vectors for the two collectors (*Pc1_b*, *Pc1_e*, *Pc2_b*, and *Pc2_e*), and the velocity vectors for the emitter (*Ve*) and collectors (*Vc1* and *Vc2*). The function then computes the beginning and ending TDOAs and FDOAs by implementing Equations (4-8) and (4-15), respectively. The results are then displayed in the MATLAB command window.

B. EXAMPLE GEOMETRIES AND SIGNAL SETS

In order to test the sig_gen.m function and ensure that it operates correctly, several different signal sets were generated with the software, and then input into CAF.m to compare actual TDOA and FDOA measurements with the theoretical values calculated by tdoa_fdoa.m. The following subsections document the results of five different pairs of signals. The first three show the results of simple geometries that produce different combinations of constant and time-varying TDOAs and FDOAs. The last two subsections show the results of simulating real-world geometries with spaceborne and airborne collectors.

1. Constant TDOA and Zero FDOA

As pointed out in section III.C.2, it is impossible for an emitter-collector geometry to produce simultaneously constant, non-zero TDOAs and FDOAs, because a constant TDOA causes the associated FDOA to be zero. To illustrate this point, a pair of signals was generated with the parameters and geometry inputs listed in Figure (5-1), which is the MATLAB command window after running the sig_gen.m function. The geometry is such that the emitter and both colletors are on a line in the x-direction. Both collectors are to the right of, and moving away from, the emitter. Note that in MATLAB, “1e5” is mathematically equivalent to $1x10^5$. Also note that the defined geometry and velocities for this example are not necessarily realistic. They just represent a simple case showing that a constant TDOA leads to an FDOA of zero. Finally, notice that the sampling frequency is less than the carrier frequency. This does not adversely impact the CAF computations since the goal is to find the TDOA and FDOA, not to preserve the

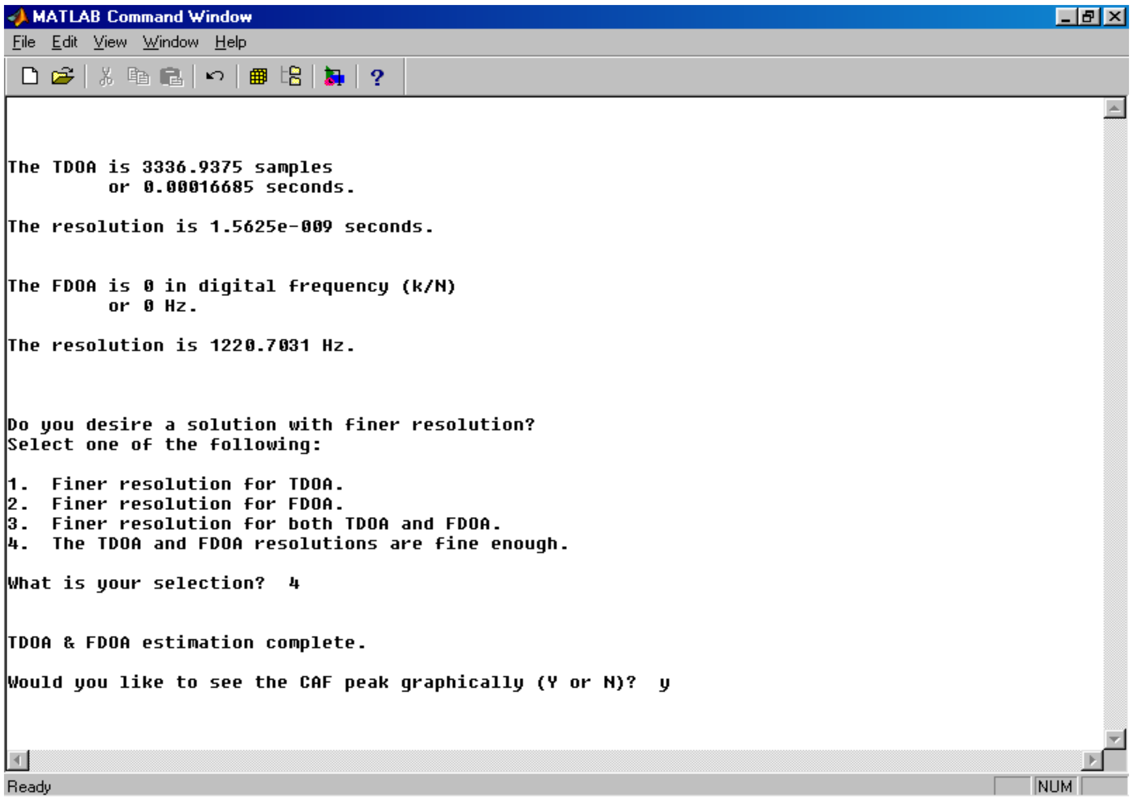


```
>> [Sa1, Sa2, S1, S2] = sig_gen;

All positions and velocites must be entered in vector format,
e.g., [X Y Z] or [X, Y, Z] (including the brackets).

Collector 1's POSITION Vector at time 0 (in meters)?  [1e4 0 0]
Collector 1's VELOCITY Vector (in m/s)?  [1e5 0 0]

Collector 2's POSITION Vector at time 0 (in meters)?  [6e4 0 0]
Collector 2's VELOCITY Vector (in m/s)?  [1e5 0 0]

Emitter's POSITION Vector at time 0 (in meters)?  [0 0 0]
Emitter's VELOCITY Vector (in m/s)?  [0 0 0]

Carrier Frequency (in Hz)?  55e6
Sampling Frequency (in Hz)?  20e6
Symbol Rate (in symbols/s)?  8e5

How many samples?  8192

Desired Es/No at Collector 1 (in dB)?  10
Desired Es/No at Collector 2 (in dB)?  10



At the START of the Collection, TDOA = 0.00016678 seconds.
                                FDOA = 0 Hertz.

At the END of the Collection, TDOA = 0.00016678 seconds.
                              FDOA = 0 Hertz.
>>
```

Figure 5-1. Example Signal Set (Constant TDOA, FDOA = 0).

integrity of the transmitted signal. It is important, however, to ensure that the sampling frequency is significantly higher than the signals' data rate. Since a BPSK signal's null-to-null bandwidth is about twice the data rate [6], sampling at the Nyquist rate (i.e., twice the bandwidth) will ensure this condition is met.

At the bottom of Figure (5-1), the theoretical TDOAs and FDOAs at the beginning and end of the collection are shown. The TDOA is 0.00016678 seconds at the beginning and end, so it is indeed constant. The theoretical FDOA is zero at both the beginning and end, as expected. Figure (5-2) shows the MATLAB command window results when the signal pair is input into CAF.m. After a few iterations of the fine mode, CAF.m calculated the TDOA as 0.00016685 seconds, which has an error of about 0.042 percent of the theoretical calculation. And the measured FDOA is exactly zero. Figures (5-3) and (5-4) show the 3-D and 2-D views of the CAF surface, respectively. Note that the surface has a very well defined peak that is narrow and nearly triangular. This is to



```


The TDOA is 3336.9375 samples
         or 0.00016685 seconds.

The resolution is 1.5625e-009 seconds.


The FDOA is 0 in digital frequency (k/N)
         or 0 Hz.

The resolution is 1220.7031 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 5-2. CAF.m Results (Constant TDOA, FDOA = 0).

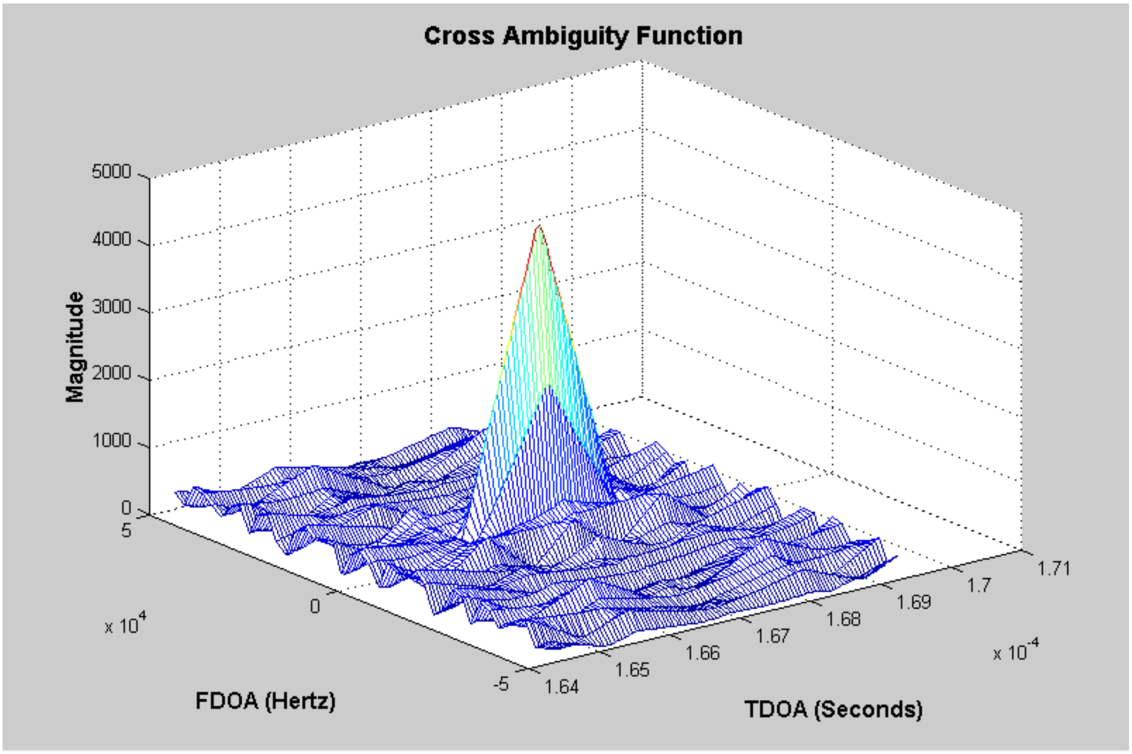


Figure 5-3. 3-D CAF Surface (Constant TDOA, FDOA = 0).

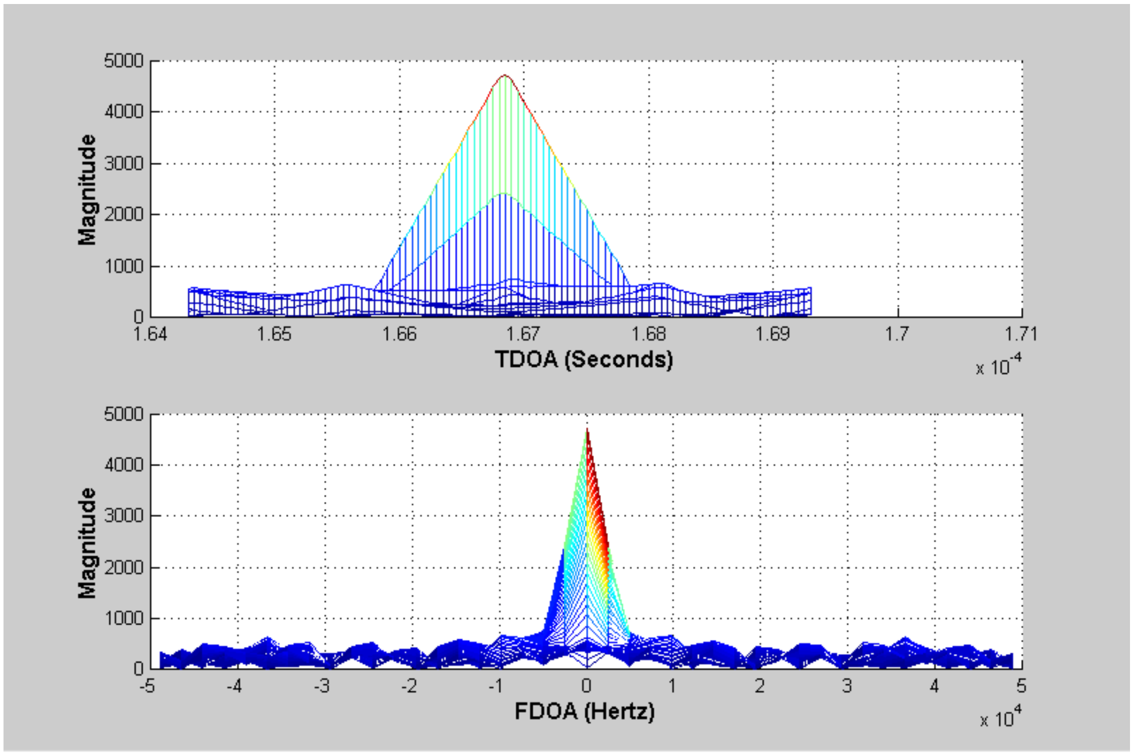
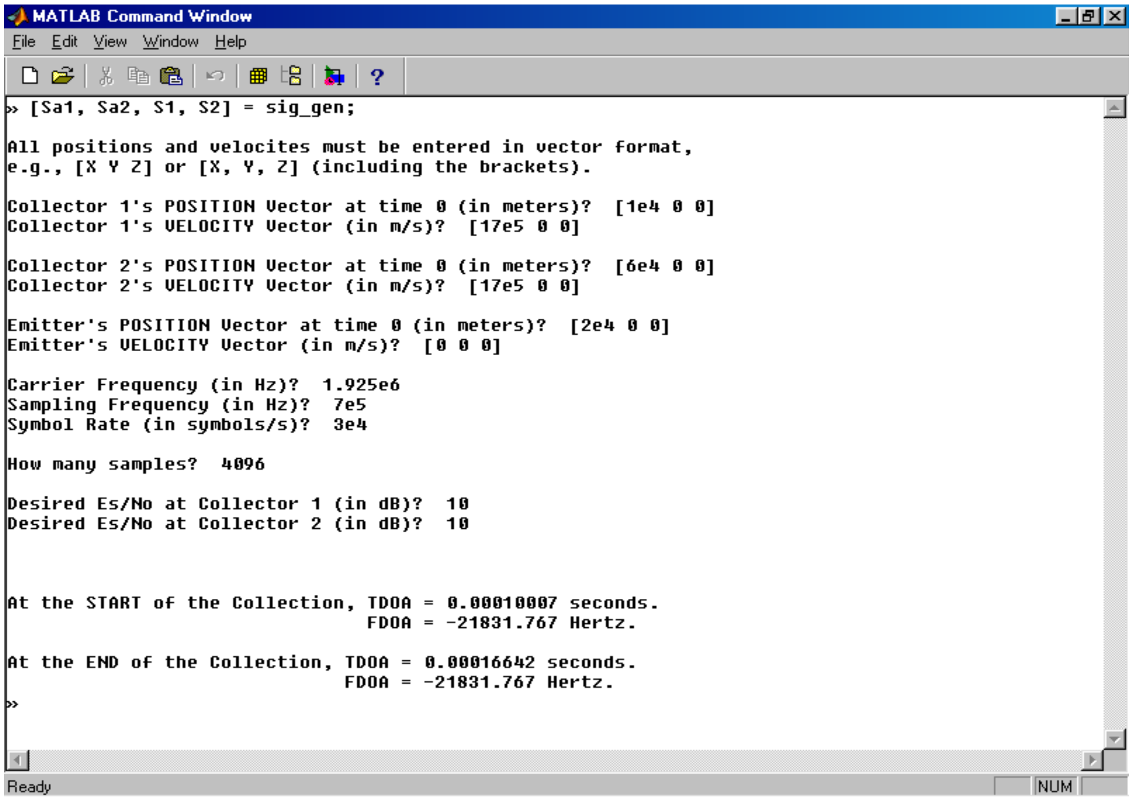


Figure 5-4. 2-D Cuts Through CAF Surface (Constant TDOA, FDOA = 0).

be expected for signals with rectangular envelopes, and for situations where the TDOA is constant. The next section will show how a time-varying TDOA affects the resulting CAF surface.

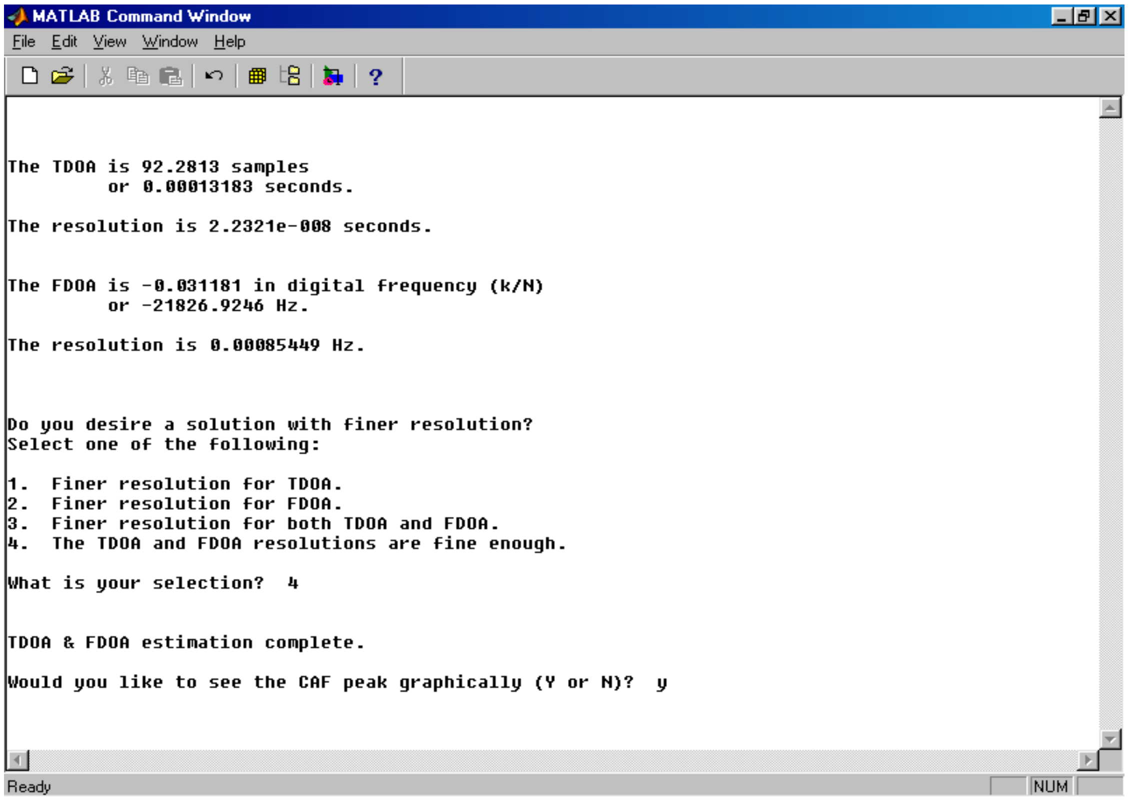
2. Time-Varying TDOA and Constant FDOA

Figure (5-5) shows the geometry and parameters used to generate a pair of signals that has a time-varying TDOA and a constant FDOA. Again, the geometry and signal parameters are not at all realistic, but the values were chosen in order to exaggerate the effect that a time-varying TDOA has on the CAF surface. For comparison purposes, the values were also chosen so that the ending TDOA would be somewhat close to the constant TDOA of the previous section. As Figure (5-5) shows, the TDOA is clearly time-varying, since it is 0.00010007 seconds at the beginning of the collection and 0.00016642 seconds at the end. The FDOA is a constant value equal to –21831.767 Hz.



```
>> [Sa1, Sa2, S1, S2] = sig_gen;

All positions and velocites must be entered in vector format,
e.g., [X Y Z] or [X, Y, Z] (including the brackets).

Collector 1's POSITION Vector at time 0 (in meters)?  [1e4 0 0]
Collector 1's VELOCITY Vector (in m/s)?  [17e5 0 0]

Collector 2's POSITION Vector at time 0 (in meters)?  [6e4 0 0]
Collector 2's VELOCITY Vector (in m/s)?  [17e5 0 0]

Emitter's POSITION Vector at time 0 (in meters)?  [2e4 0 0]
Emitter's VELOCITY Vector (in m/s)?  [0 0 0]

Carrier Frequency (in Hz)?  1.925e6
Sampling Frequency (in Hz)?  7e5
Symbol Rate (in symbols/s)?  3e4

How many samples?  4096

Desired Es/No at Collector 1 (in dB)?  10
Desired Es/No at Collector 2 (in dB)?  10


At the START of the Collection, TDOA = 0.00010007 seconds.
                                FDOA = -21831.767 Hertz.

At the END of the Collection, TDOA = 0.00016642 seconds.
                              FDOA = -21831.767 Hertz.
>>
```

Figure 5-5. Example Signal Set (Time-Varying TDOA, Constant FDOA).



```


The TDOA is 92.2813 samples
         or 0.00013183 seconds.

The resolution is 2.2321e-008 seconds.


The FDOA is -0.031181 in digital frequency (k/N)
         or -21826.9246 Hz.

The resolution is 0.00085449 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 5-6. CAF.m Results (Time-Varying TDOA, Constant FDOA).

Figure (5-6) shows the MATLAB command window that results from running the signals through CAF.m and iterating the fine calculations a few times. The TDOA is calculated to be 0.00013183 seconds. Since the TDOA is time-varying, the only check for validity is that the computed TDOA is indeed somewhere between the theoretical values of the TDOA at the beginning and end of the collect. The computed value of the FDOA is –21826.969 Hz, which is about 0.022 percent different from the theoretical value. Figures (5-7) and (5-8) show the resulting CAF surface in 3-D and 2-D, respectively. The most striking difference between this surface and the one in the previous section is that this peak is nowhere near triangular. It is much broader and flatter than the previous surface. This result makes perfect sense. After all, if a TDOA is constant, then the resulting surface should present a very specific and well-defined peak. If the TDOA is time-varying, on the other hand, the resulting surface will have a peak whose only requirement is that it fall within the possible range of TDOAs defined for the

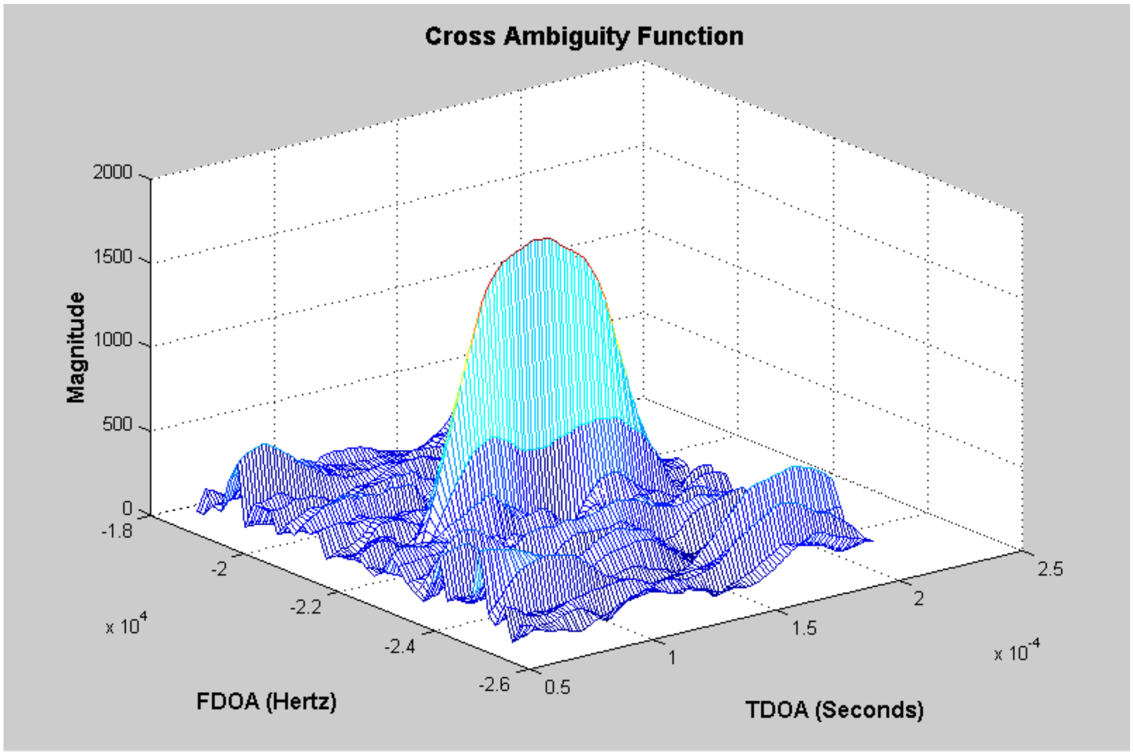


Figure 5-7. 3-D CAF Surface (Time-Varying TDOA, Constant FDOA).

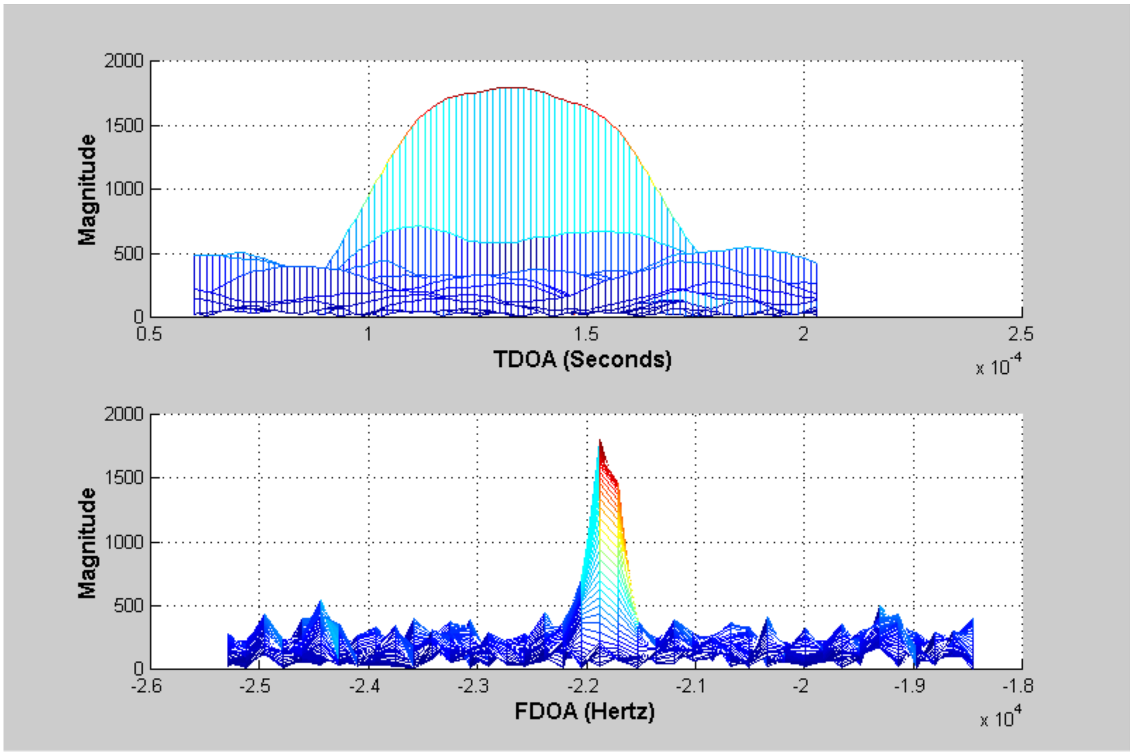


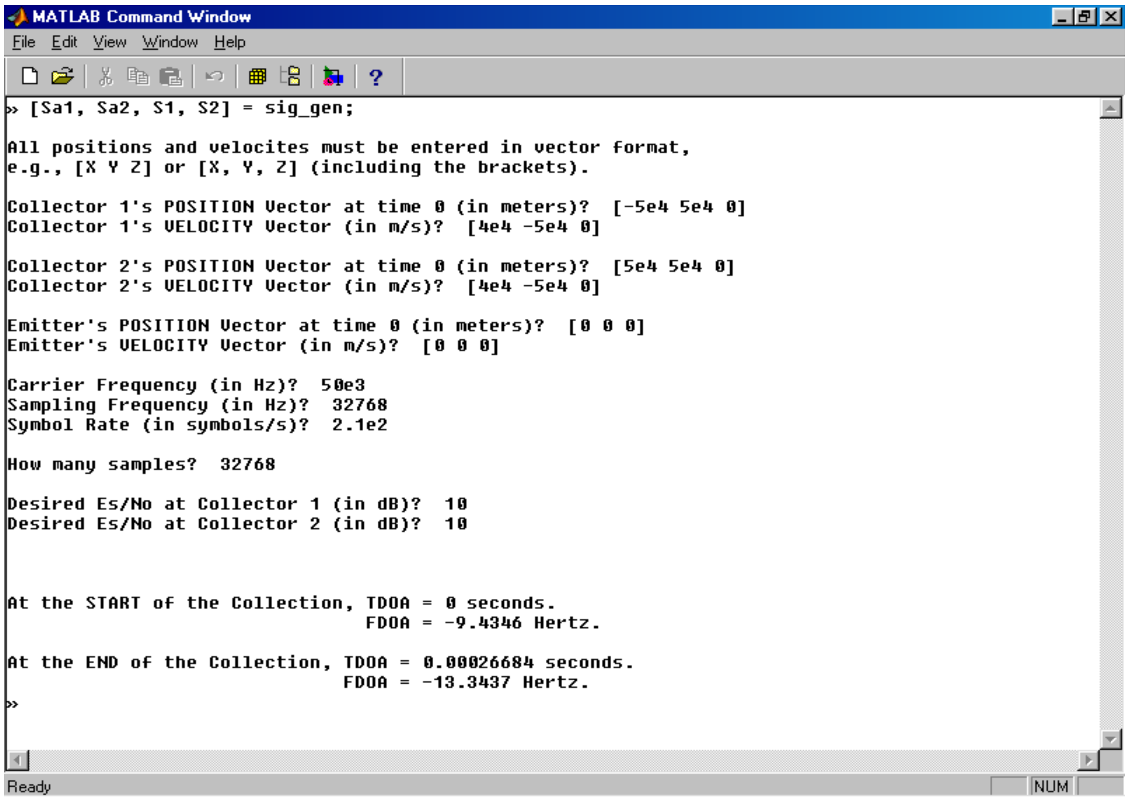
Figure 5-8. 2-D Cuts Through CAF Surface (Time-Varying TDOA, Constant FDOA).

duration of collection. In the next section, a pair of signals with time-varying TDOA and time-varying FDOA is presented.

3. Time-Varying TDOA and Time-Varying FDOA

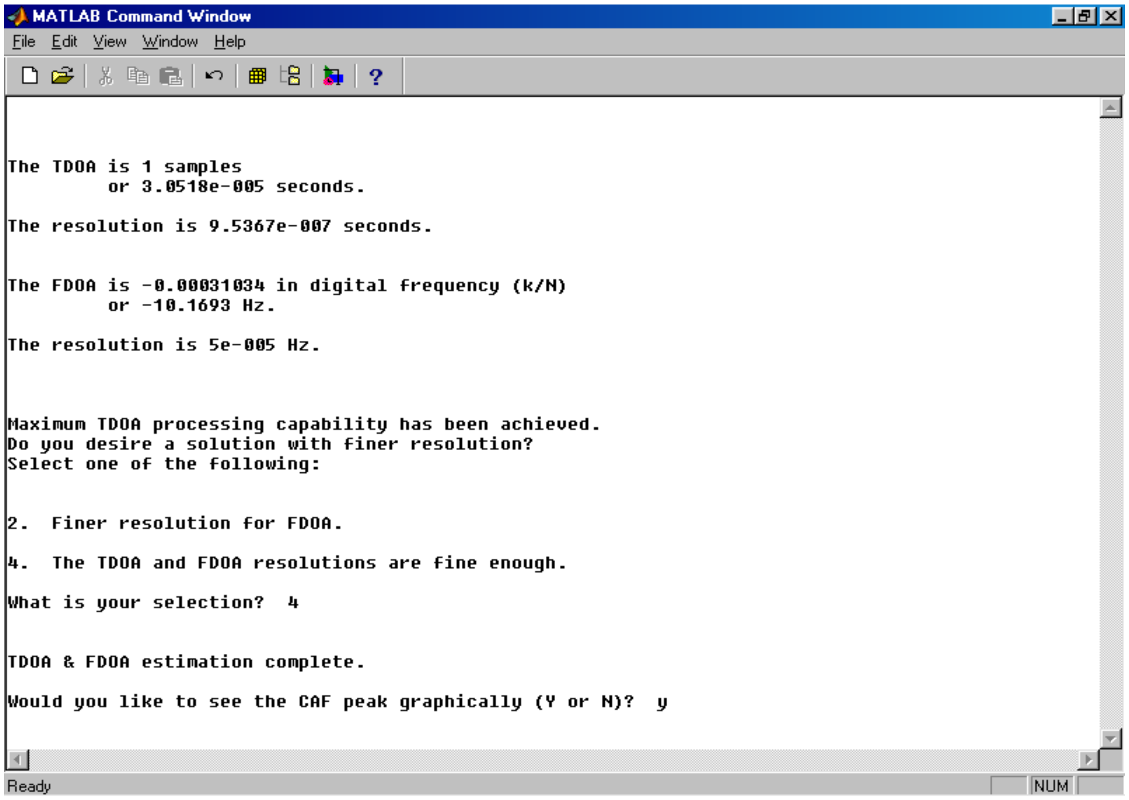
Figure (5-9) shows the parameters and geometry for a pair of generated signals with a TDOA and FDOA that are both time-varying. This is clear from the theoretical calculations. The TDOA goes from 0 to 0.00026684 seconds, and the FDOA goes from –9.4346 to –13.3437 Hz. Figure (5-10) shows the results of inputting the signals into CAF.m. After a few iterations, the TDOA is computed as $3.0518x10^{-5}$ seconds and the FDOA is computed as –10.1693 Hz. Both of these values are within the ranges of the theoretical values.

Figures (5-11) and (5-12) show the resulting CAF surface. Notice that the peak is fairly wide along the TDOA axis, and that it is also wide along the FDOA axis. This confirms the fact that time-varying TDOAs and FDOAs cause the peak of a CAF surface



```
» [Sa1, Sa2, S1, S2] = sig_gen;

All positions and velocites must be entered in vector format,
e.g., [X Y Z] or [X, Y, Z] (including the brackets).

Collector 1's POSITION Vector at time 0 (in meters)?  [-5e4 5e4 0]
Collector 1's VELOCITY Vector (in m/s)?  [4e4 -5e4 0]

Collector 2's POSITION Vector at time 0 (in meters)?  [5e4 5e4 0]
Collector 2's VELOCITY Vector (in m/s)?  [4e4 -5e4 0]

Emitter's POSITION Vector at time 0 (in meters)?  [0 0 0]
Emitter's VELOCITY Vector (in m/s)?  [0 0 0]

Carrier Frequency (in Hz)?  50e3
Sampling Frequency (in Hz)?  32768
Symbol Rate (in symbols/s)?  2.1e2

How many samples?  32768

Desired Es/No at Collector 1 (in dB)?  10
Desired Es/No at Collector 2 (in dB)?  10



At the START of the Collection, TDOA = 0 seconds.
                                FDOA = -9.4346 Hertz.

At the END of the Collection, TDOA = 0.00026684 seconds.
                              FDOA = -13.3437 Hertz.
»
```

Figure 5-9. Example Signal Set (Time-Varying TDOA and FDOA).



```


The TDOA is 1 samples
         or 3.0518e-005 seconds.

The resolution is 9.5367e-007 seconds.


The FDOA is -0.00031034 in digital frequency (k/N)
         or -10.1693 Hz.

The resolution is 5e-005 Hz.



Maximum TDOA processing capability has been achieved.
Do you desire a solution with finer resolution?
Select one of the following:


2.  Finer resolution for FDOA.

4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 5-10. CAF.m Results (Time-Varying TDOA and FDOA).

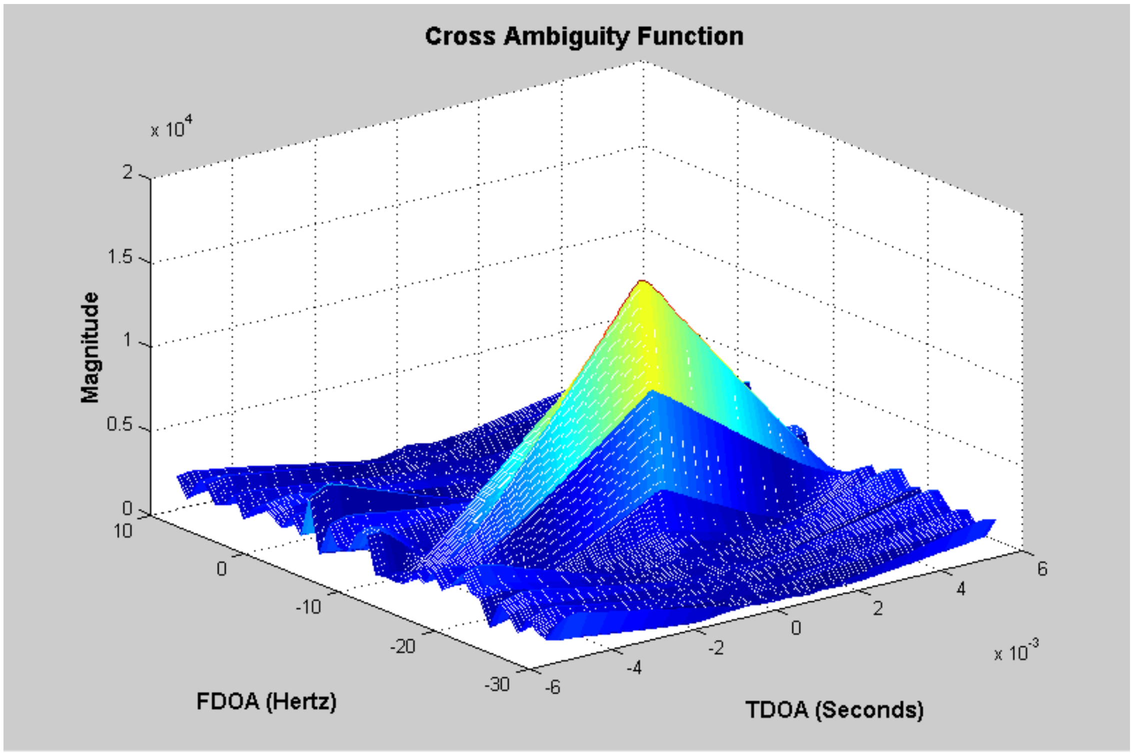


Figure 5-11. 3-D CAF Surface (Time-Varying TDOA and FDOA).

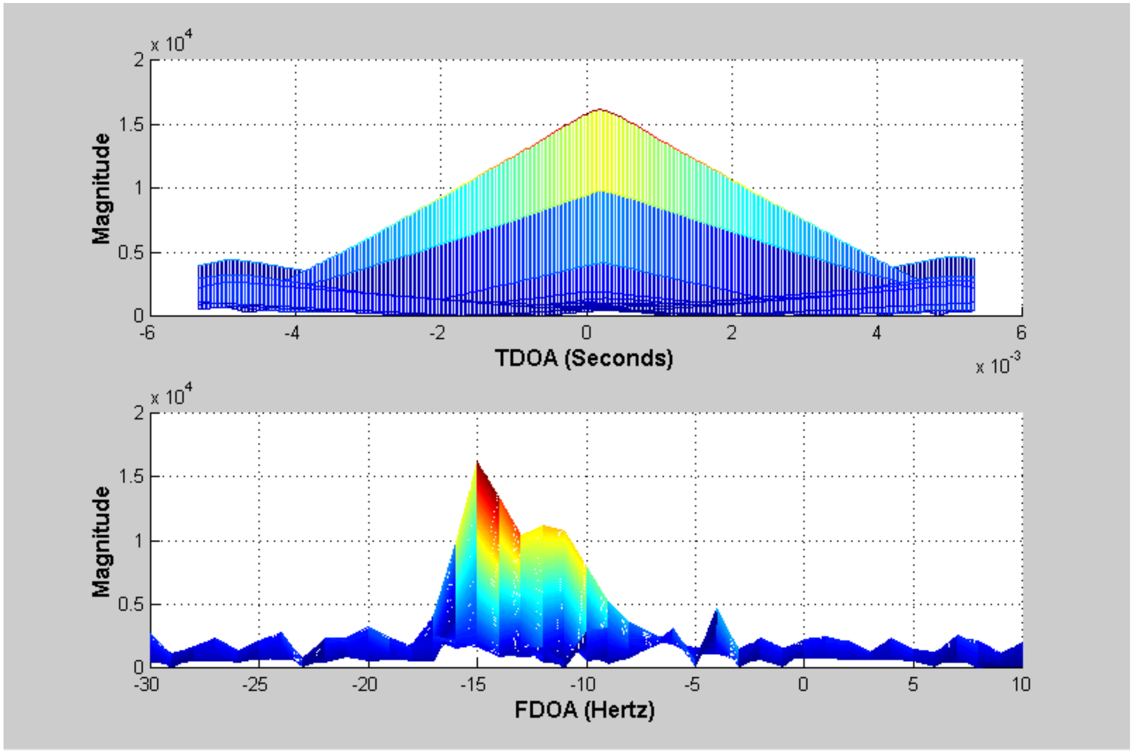


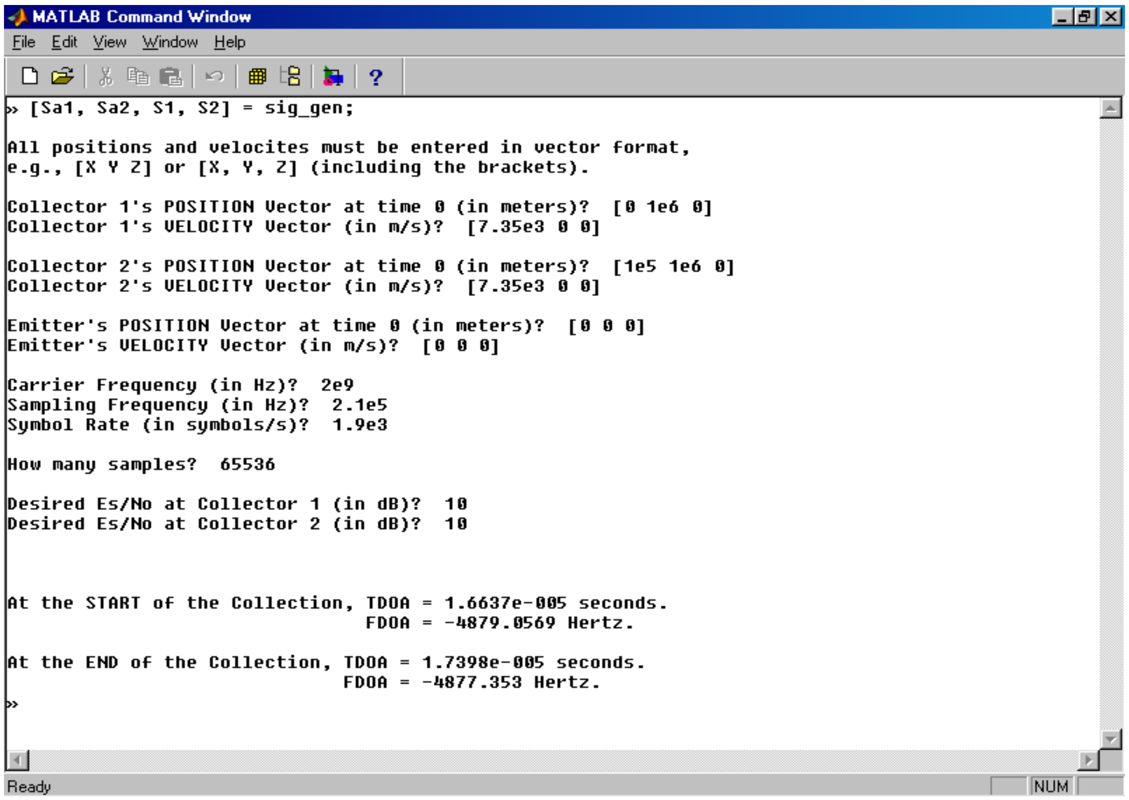
Figure 5-12. 2-D Cuts Through CAF Surface (Time-Varying TDOA and FDOA).

to “spread.” In this case, the spreading is somewhat unusual, making the surface appear to have two ridges on the FDOA axis. Also note that the TDOA and FDOA computed by CAF.m and shown in Figure (5-10) appear to be on the smaller ridge in Figure (5-12). This can happen when multiple peaks appear in the CAF. This phenomenon is discussed further in Chapter VI.

4. Simulated Low Earth Orbit Satellite Collectors

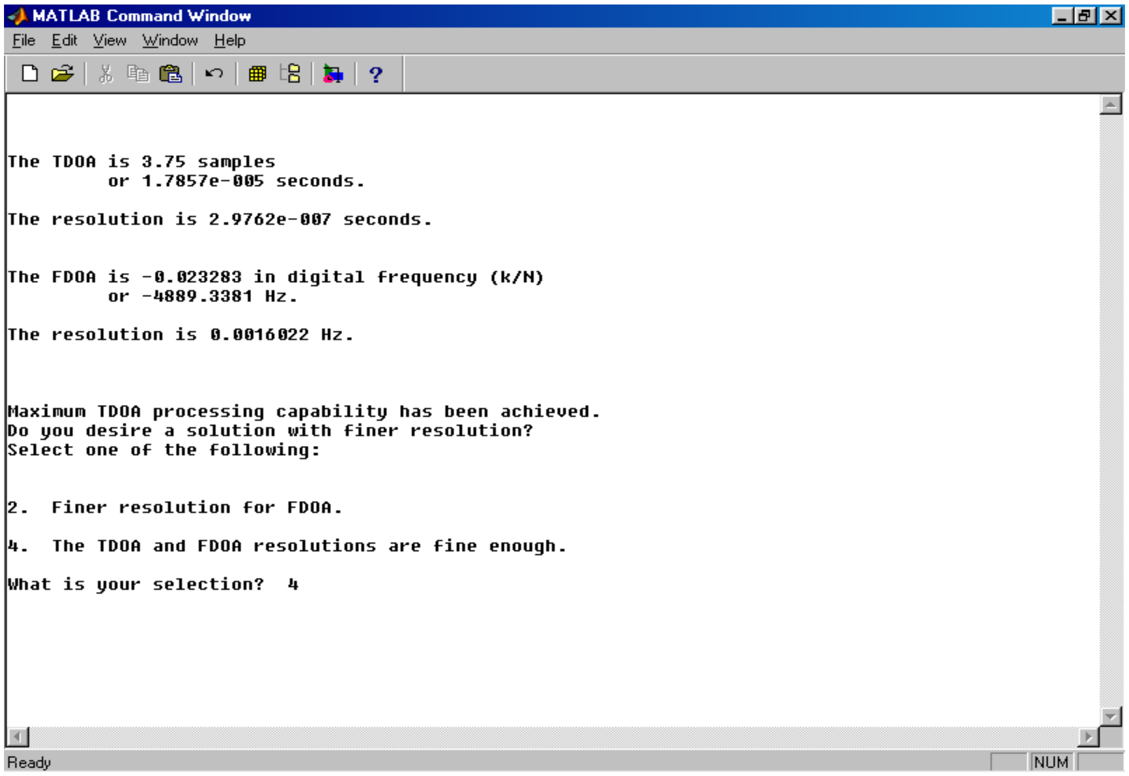
The results of the preceding three sections validate the algorithm used in the sig_gen.m since the computed TDOAs and FDOAs compared favorably with the theoretical calculations in each case. As stated before, however, the parameters and geometries used were not realistic for real-world systems. In this section, a pair of signals is generated with a realistic geometry: a stationary ground-based emitter and a pair of satellite collectors in a Low Earth Orbit (LEO) of 1000 kilometers. The collectors are spaced 100 kilometers apart on a line parallel to the earth’s surface (assumed flat). For this geometry, the orbital velocity of the collectors is 7.35 kilometers per second [12]. At time 0, collector number one is directly above the emitter on the earth’s surface. The carrier frequency of the signal is 2 GHz. Many test cases showed that defining sampling frequency, carrier frequency, and/or symbol rate such that they are exact multiples of each other produces unreliable results. Accordingly, the sampling frequency is set to 0.21 MHz and the symbol rate is 1900 symbols per second. Note that the sampling frequency is three orders of magnitude below the carrier frequency, but it is much greater than the data rate, as required. Figure (5-13) shows the result of running the sig_gen.m software for the situation described above. Notice that the theoretical TDOA and FDOA values indicate that both are time-varying, although not by much. This is because the relatively short collection time does not produce a large disparity in the geometry at the beginning and end of the collection.

Figure (5-14) shows that the CAF.m’s computation of the TDOA and FDOA compares favorably with the theoretical values. Figures (5-15) and (5-16) show the resulting CAF surface. The peak of the surface does not come to an exact point, so a



```
» [Sa1, Sa2, S1, S2] = sig_gen;

All positions and velocites must be entered in vector format,
e.g., [X Y Z] or [X, Y, Z] (including the brackets).

Collector 1's POSITION Vector at time 0 (in meters)?  [0 1e6 0]
Collector 1's VELOCITY Vector (in m/s)?  [7.35e3 0 0]

Collector 2's POSITION Vector at time 0 (in meters)?  [1e5 1e6 0]
Collector 2's VELOCITY Vector (in m/s)?  [7.35e3 0 0]

Emitter's POSITION Vector at time 0 (in meters)?  [0 0 0]
Emitter's VELOCITY Vector (in m/s)?  [0 0 0]

Carrier Frequency (in Hz)?  2e9
Sampling Frequency (in Hz)?  2.1e5
Symbol Rate (in symbols/s)?  1.9e3

How many samples?  65536

Desired Es/No at Collector 1 (in dB)?  10
Desired Es/No at Collector 2 (in dB)?  10



At the START of the Collection, TDOA = 1.6637e-005 seconds.
                                FDOA = -4879.0569 Hertz.

At the END of the Collection, TDOA = 1.7398e-005 seconds.
                              FDOA = -4877.353 Hertz.
»
```

Figure 5-13. Example Signal Set (LEO Satellite Collectors & Ground-Based Emitter).



```
The TDOA is 3.75 samples
         or 1.7857e-005 seconds.

The resolution is 2.9762e-007 seconds.


The FDOA is -0.023283 in digital frequency (k/N)
         or -4889.3381 Hz.

The resolution is 0.0016022 Hz.



Maximum TDOA processing capability has been achieved.
Do you desire a solution with finer resolution?
Select one of the following:


2.  Finer resolution for FDOA.

4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4
```

Figure 5-14. CAF.m Results (LEO Satellite Collectors & Ground-Based Emitter).

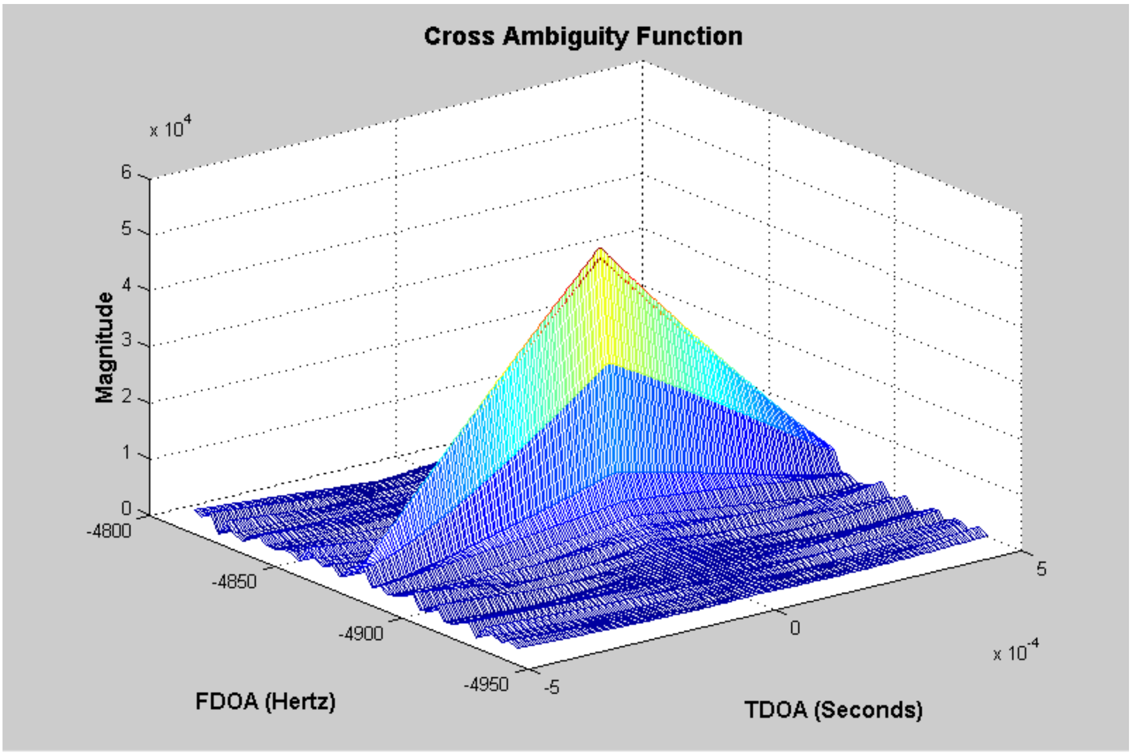


Figure 5-15. 3-D CAF Surface (LEO Satellite Collectors & Ground-Based Emitter).

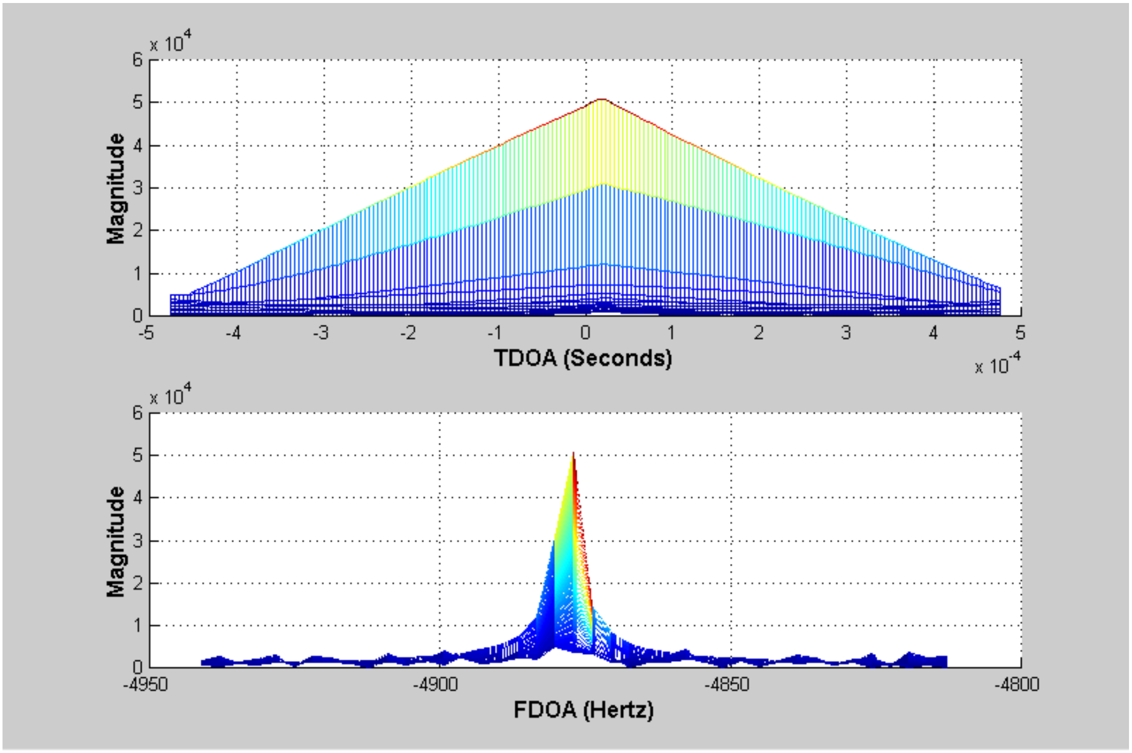


Figure 5-16. 2-D Cuts Through CAF Surface (LEO Collectors & Ground Emitter).

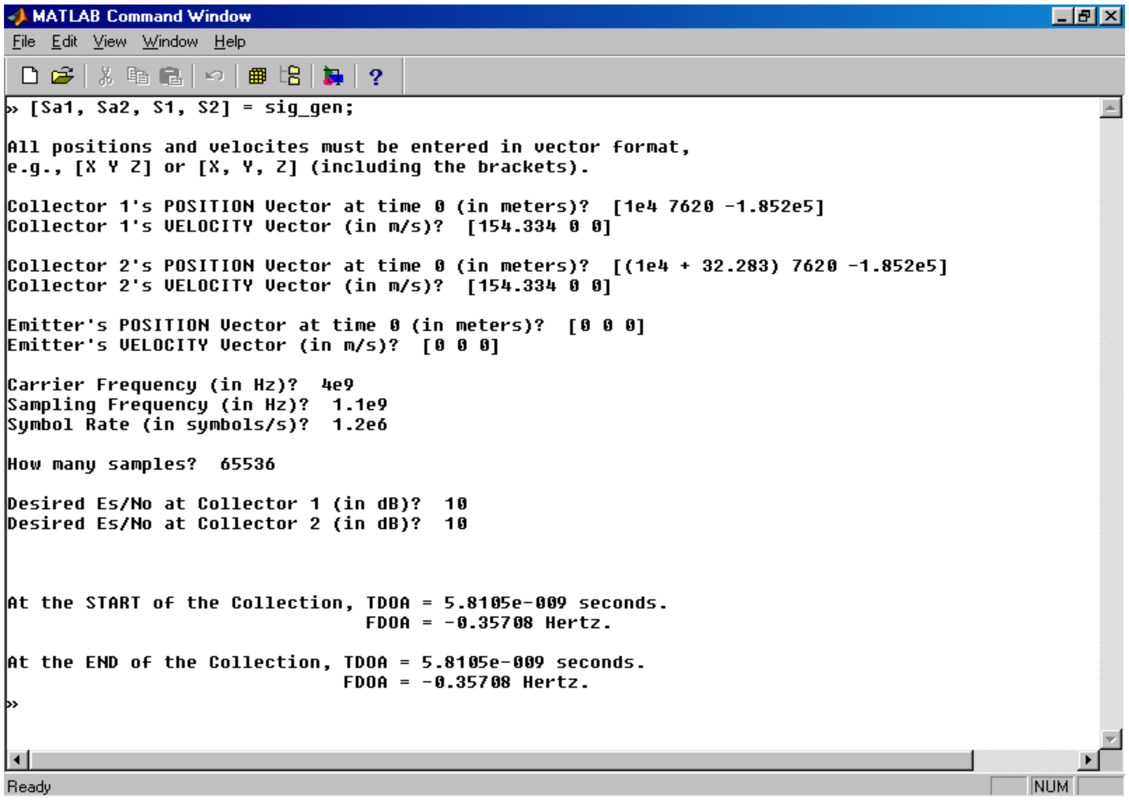
small amount of smearing is evident. On the FDOA axis in Figure (5-16), the slope of the surface on the left hand side is not quite as steep as the slope on the right hand side. This indicates a slight smearing to the left, which makes sense given the range of FDOAs predicted in Figure (5-14).

5. Simulated Airborne Collectors

In this section, another real-world system is simulated: a pair of airborne collectors and a ground-based emitter. The collectors are assumed to be mounted on an aircraft with the same dimensions and characteristics of an EP-3 Aries. One collector is assumed to be mounted in the nose of the aircraft, and the other collector in the tail. This provides for maximum separation of the collectors onboard the aircraft. From [13], the length of an EP-3 is 105 feet, 11 inches; its maximum speed is 473 knots; and its maximum altitude is 28,000 feet. For purposes of collection, the assumed speed is 300 knots, and the assumed altitude is 25,000 feet. The airplane is assumed to be flying parallel to a coastline at a distance of 100 nautical miles from the coast.

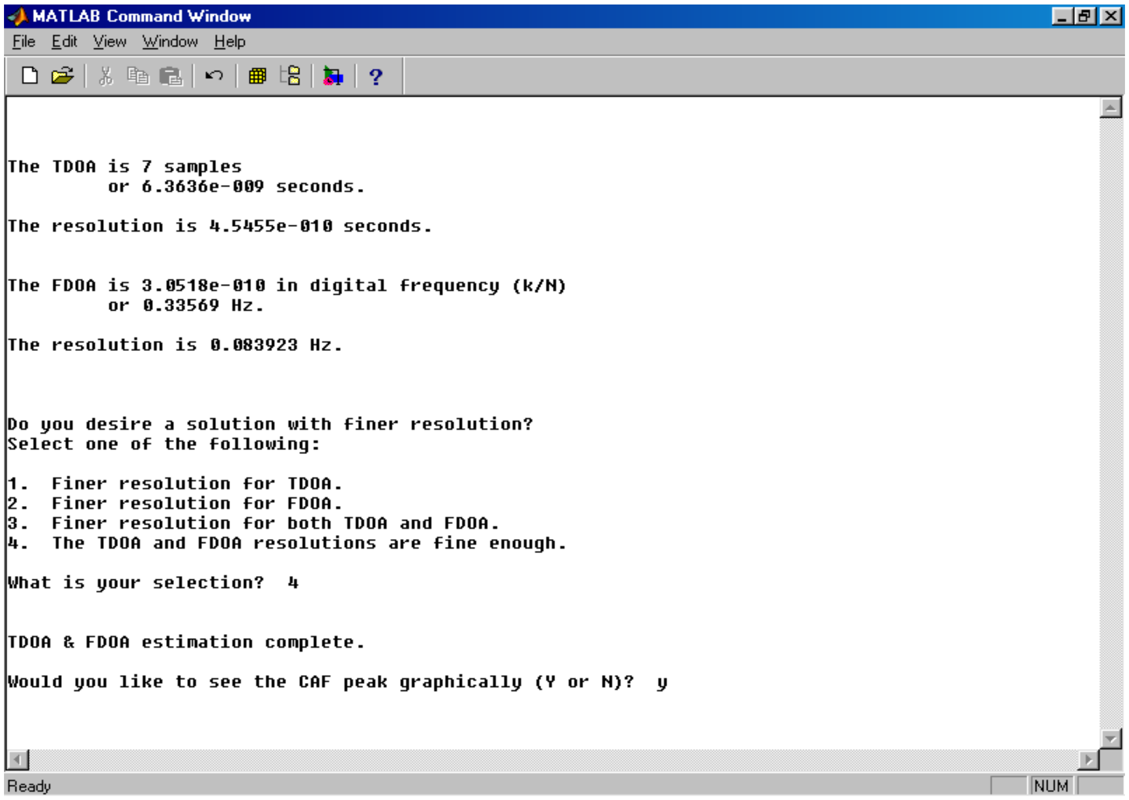
Figure (5-17) shows the results of running sig_gen.m for the situation described above. Note that all geometric inputs have been converted to meters and meters per second, as appropriate. For the Cartesian coordinate system in this case, the x-direction is parallel to the coastline, the y-direction is altitude, and the z-direction is perpendicular to the coastline in the plane of the earth's surface (i.e., it measures lateral distance from the coastline). The signal has a carrier frequency of 4 GHz and a symbol rate of 1.2 megabits per second, and it is sampled at 1.1 GHz. Notice that the resulting theoretical values of TDOA and FDOA are constant. This is only because the geometry does not change much during such a small collection time. If a much larger number of samples were taken, the TDOA and FDOA would show more change. Also notice that the FDOA, at –0.35708 Hz is a miniscule fraction of the sampling frequency!

Figure (5-18) shows the results obtained by running CAF.m. The computed TDOA and FDOA compare reasonably well with the theoretical values. The computed FDOA differs from its theoretical value by a minus sign, or about 0.7 Hz total. Although this is a large error percentage-wise, it is reasonable when considering that the FDOA



```
» [Sa1, Sa2, S1, S2] = sig_gen;

All positions and velocites must be entered in vector format,
e.g., [X Y Z] or [X, Y, Z] (including the brackets).

Collector 1's POSITION Vector at time 0 (in meters)?  [1e4 7620 -1.852e5]
Collector 1's VELOCITY Vector (in m/s)?  [154.334 0 0]

Collector 2's POSITION Vector at time 0 (in meters)?  [(1e4 + 32.283) 7620 -1.852e5]
Collector 2's VELOCITY Vector (in m/s)?  [154.334 0 0]

Emitter's POSITION Vector at time 0 (in meters)?  [0 0 0]
Emitter's VELOCITY Vector (in m/s)?  [0 0 0]

Carrier Frequency (in Hz)?  4e9
Sampling Frequency (in Hz)?  1.1e9
Symbol Rate (in symbols/s)?  1.2e6

How many samples?  65536

Desired Es/No at Collector 1 (in dB)?  10
Desired Es/No at Collector 2 (in dB)?  10



At the START of the Collection, TDOA = 5.8105e-009 seconds.
                                FDOA = -0.35708 Hertz.

At the END of the Collection, TDOA = 5.8105e-009 seconds.
                              FDOA = -0.35708 Hertz.
»
```

Figure 5-17. Example Signal Set (Airborne Collectors & Ground-Based Emitter).



```


The TDOA is 7 samples
         or 6.3636e-009 seconds.

The resolution is 4.5455e-010 seconds.


The FDOA is 3.0518e-010 in digital frequency (k/N)
         or 0.33569 Hz.

The resolution is 0.083923 Hz.



Do you desire a solution with finer resolution?
Select one of the following:

1.  Finer resolution for TDOA.
2.  Finer resolution for FDOA.
3.  Finer resolution for both TDOA and FDOA.
4.  The TDOA and FDOA resolutions are fine enough.

What is your selection?  4


TDOA & FDOA estimation complete.

Would you like to see the CAF peak graphically (Y or N)?  y
```

Figure 5-18. CAF.m Results (Airborne Collectors & Ground-Based Emitter).

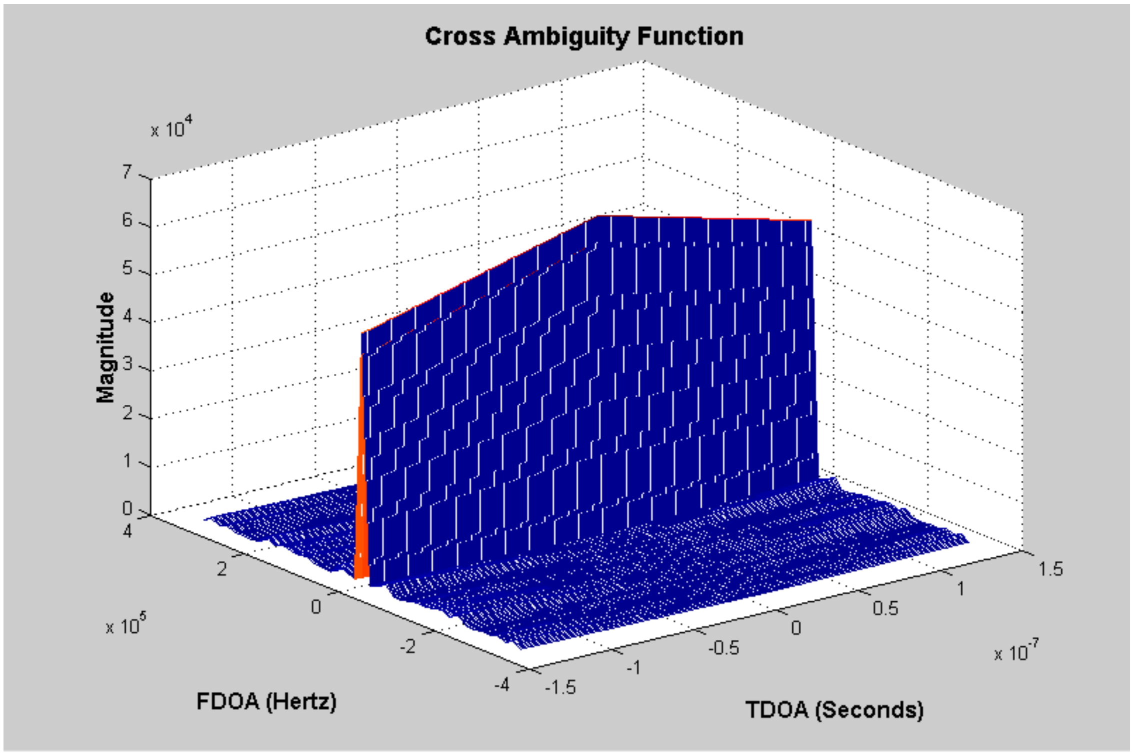


Figure 5-19. 3-D CAF Surface (Airborne Collectors & Ground-Based Emitter).

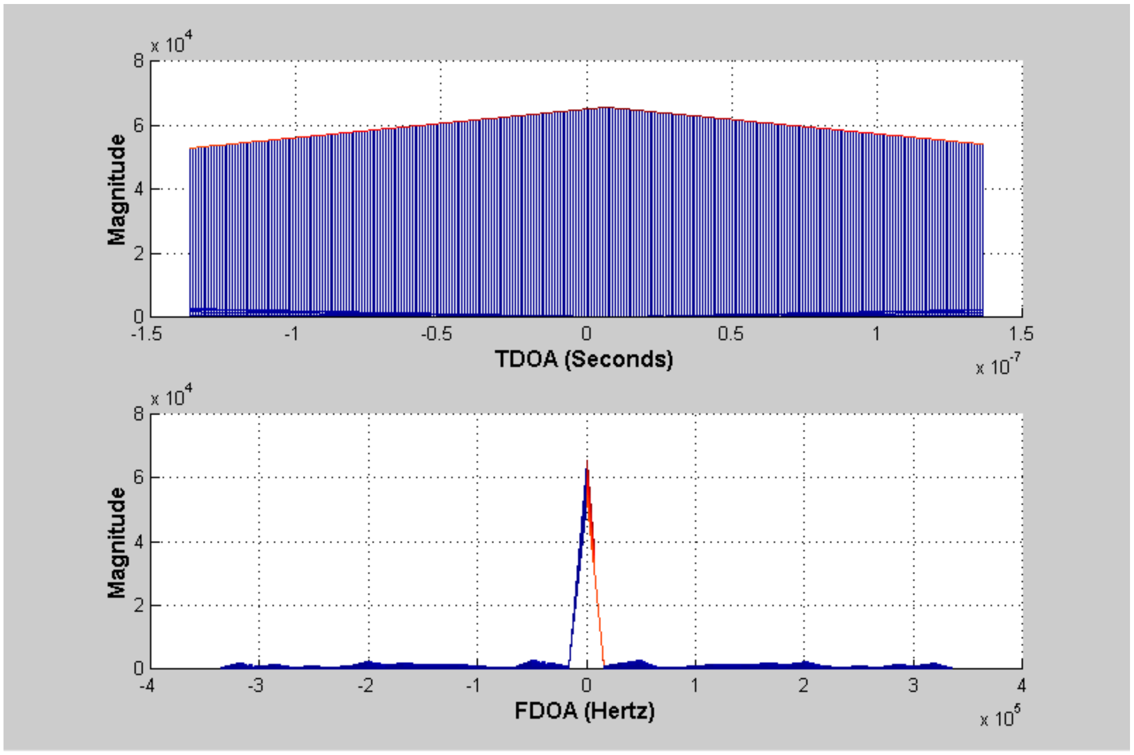


Figure 5-20. 2-D Cuts Through CAF Surface (Airborne Collectors & Ground Emitter).

resolution for N = 65536 is $\frac{1.1x10^9 Hz}{65536\, samples}$ = 16,784.7 Hz per sample! In this context, CAF.m did a reasonable job of picking out a tiny FDOA.

Figures (5-19) and (5-20) show the resulting CAF surface. The peak occurs at the expected TDOA of roughly 6 nanoseconds, but the FDOA at the peak is 0 Hz. This makes sense considering the discussion in the previous paragraph. For this situation, the FDOA is such a small fraction of the sampling frequency that the resolution achieved with N = 65536 is not sufficient to show the correct FDOA graphically. Recall that CAF_peak.m uses the FFT method to generate the CAF values and plot the surface. This is why CAF.m is able to determine the FDOA, while the resulting plot is unable to show it. This section, along with the previous one, has shown that the programs created for this thesis are indeed able to model signals with realistic parameters and emitter-collector geometries, as well as compute the embedded TDOA and FDOA with a good degree of accuracy. Section V.C below will demonstrate the CAF's ability to *detect* signals.

C. USING THE CAF FOR SIGNAL DETECTION

As shown throughout this thesis, the CAF is able to compute the TDOA and FDOA between two receivers collecting signals from the same emitter. The TDOA and FDOA information can then be used to locate the emitter. In many cases, the presence of the signal(s) may be known prior to collection. The CAF itself, however, can also be used to *detect* the presence of a signal by processing a pair of collections and looking for peaks above the noise floor. This can be useful for Low Probability of Detection (LPD) signals, which may be transmitted at very low $\frac{E_s}{N_0}$ levels. As an example, consider the LEO collector system modeled in section V.B.4 above. Using all of the parameters in Figure (5-13), the $\frac{E_s}{N_0}$ was successively reduced to show the effects on CAF computations and the CAF surface. Figures (5-21) through (5-25) show the 2-D cuts through the CAF surface for $\frac{E_s}{N_0}$ levels of –20 dB, –40 dB, –45 dB, –50 dB, and –55 dB, respectively. The –20 dB level does not appear to have affected the CAF surface much at

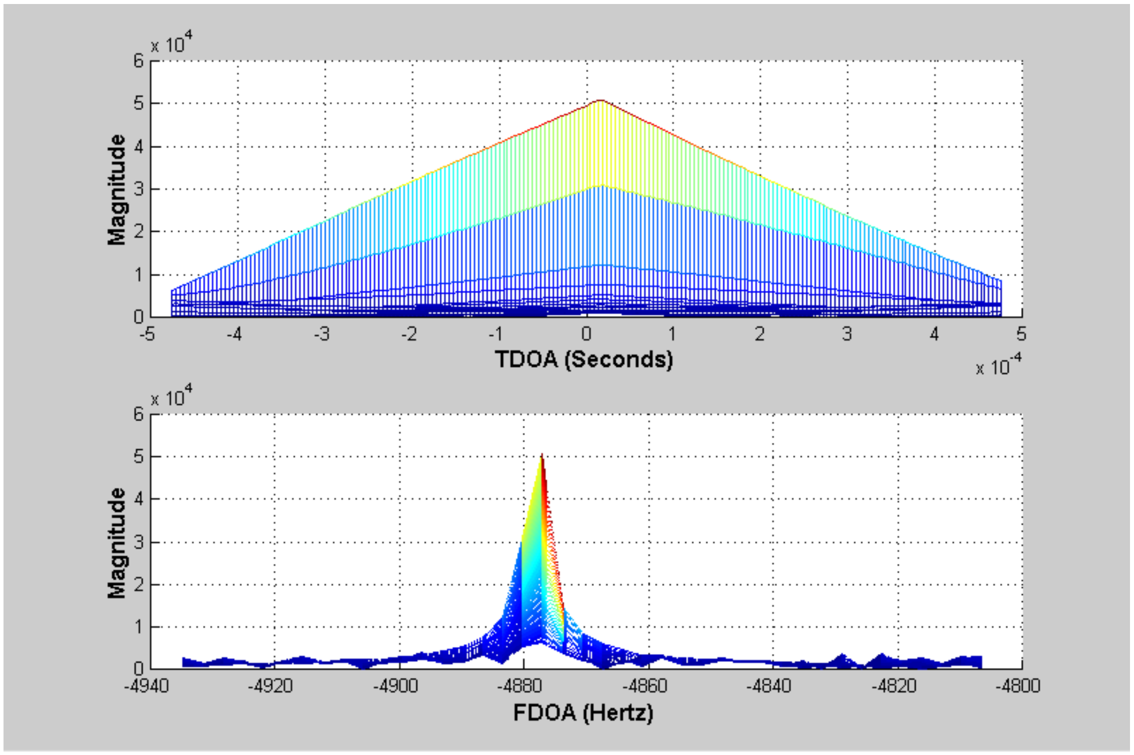


Figure 5-21. 2-D Cuts Through CAF Surface (LEO Collectors, Es/No = –20 dB).

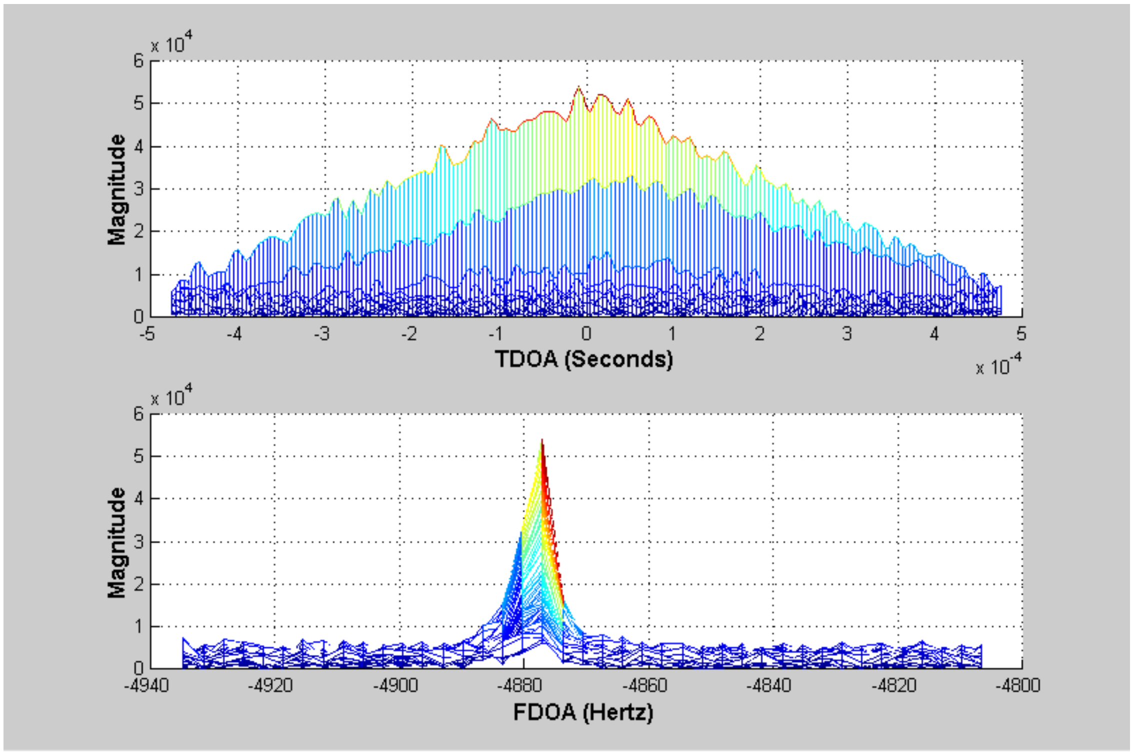


Figure 5-22. 2-D Cuts Through CAF Surface (LEO Collectors, Es/No = –40 dB).

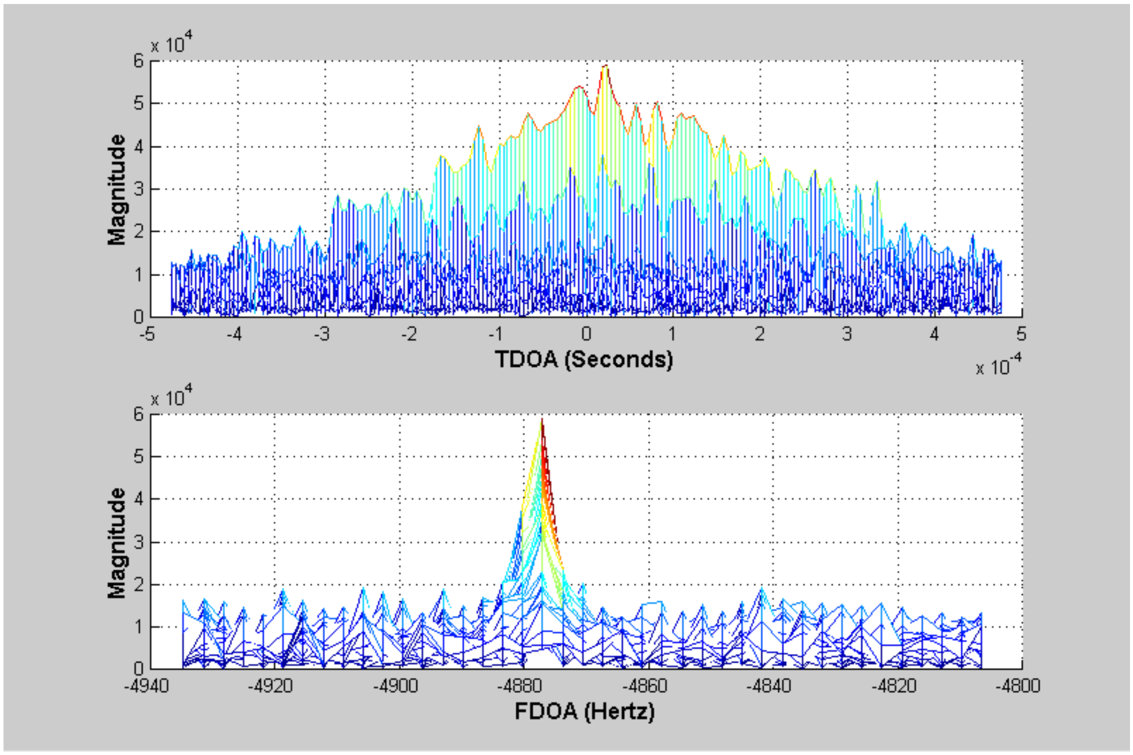


Figure 5-23. 2-D Cuts Through CAF Surface (LEO Collectors, Es/No = –45 dB).

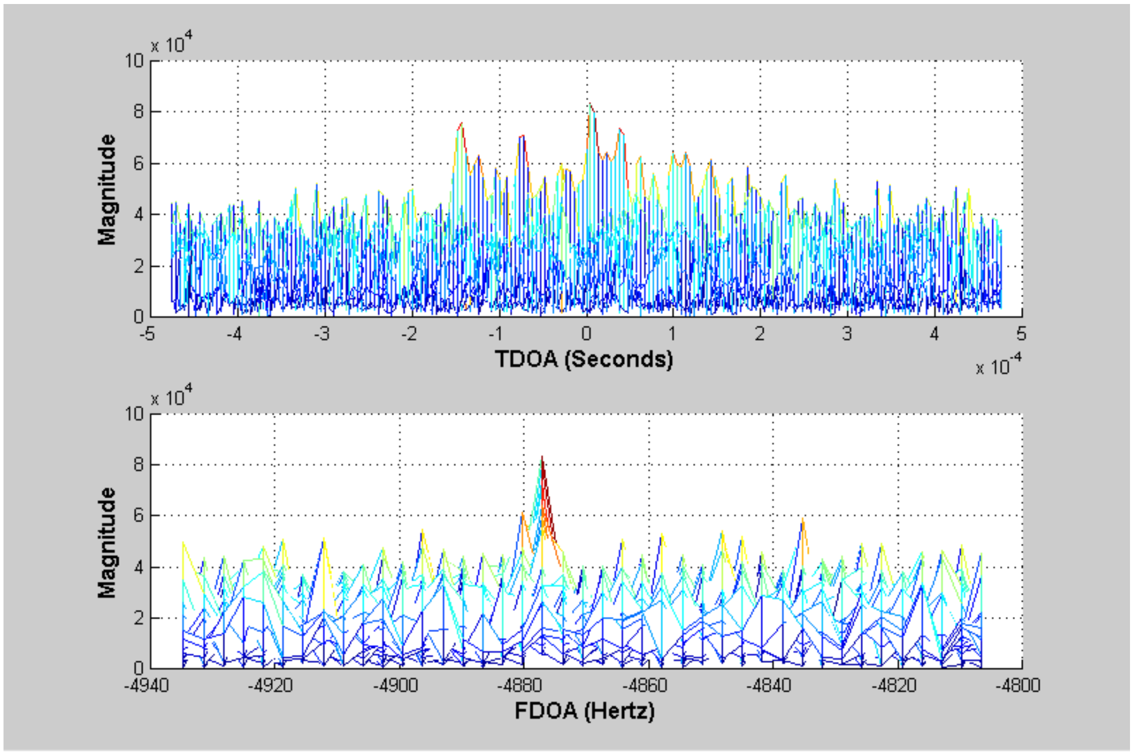


Figure 5-24. 2-D Cuts Through CAF Surface (LEO Collectors, Es/No = –50 dB).

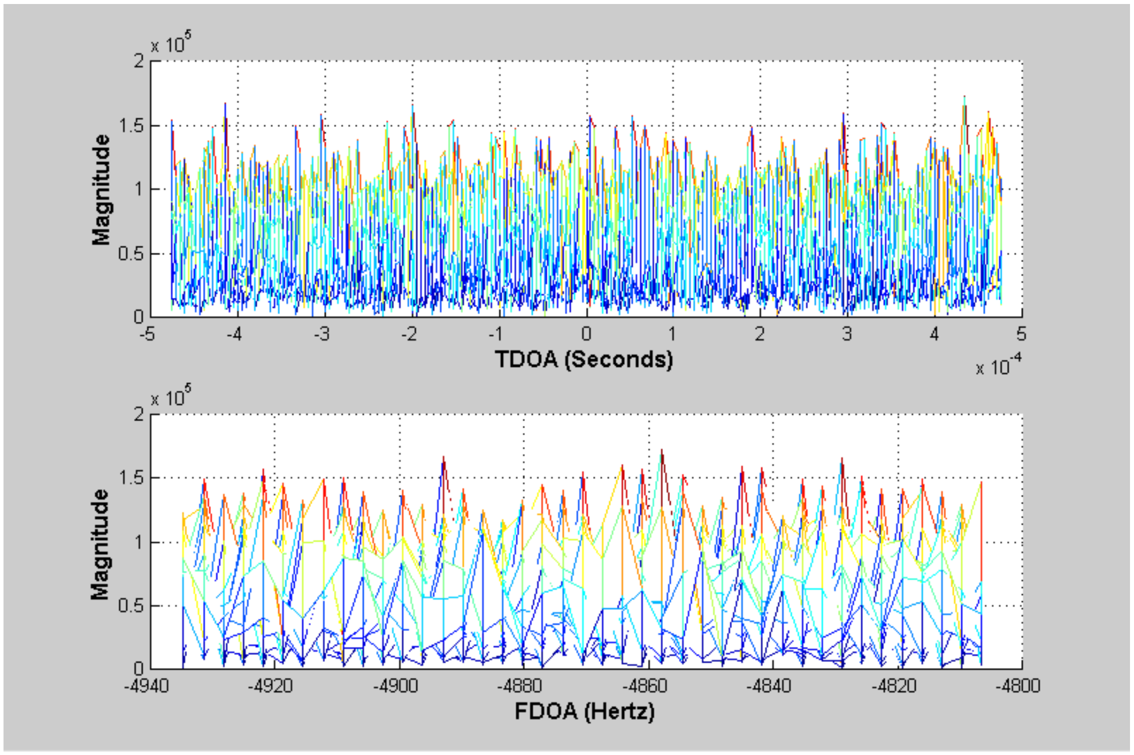


Figure 5-25. 2-D Cuts Through CAF Surface (LEO Collectors, Es/No = –55 dB).

all. When the level is reduced to –40 dB, however, the surface is noticeable deformed. At –45 dB, the noise floor is getting noticeably higher compared to the peak. At –50 dB, the surface is near the point of undetectability. At –55 dB, the noise has completely buried the surface. Although the CAF.m results are not depicted here, the program computed reasonable TDOAs and FDOAs up through the –45 dB level. Thus, the CAF is able to detect the presence of the signal, and to estimate the TDOA and FDOA for subsequent location of the emitter. Below –45 dB, however, the numerical results were completely incorrect. Of course, if detecting a signal is the priority (as opposed to locating the emitter), then the TDOA and FDOA are not crucial. A relatively good estimation of TDOA and FDOA may be meaningless anyway, considering the jagged edges on the CAF surface. In any case, the existence of any surface clearly above the noise floor indicates the presence of a signal. If no signal were present, then the CAF would be processing nothing but noise, and the plot would reflect just that. Ideally, to detect the presence of a signal, one should know at least the carrier frequency and a rough idea of where the emitter is in order to narrow the “search” range for the CAF.

In this chapter the sig_gen.m program was described in detail. A number of signal sets were generated and input into CAF.m to ensure that the resulting TDOAs and FDOAs compared favorably with theoretical values. The results confirmed that the sig_gen.m program functions properly. Two realistic signal sets were generated that modeled practical emitter-collector geometries: one with LEO satellite collectors and one with airborne collectors. This showed that sig_gen.m provides a very useful capability to simulate signals for real-world situations. Chapter VI will summarize the research and work done on this thesis, as well as provide some ideas for future work in CAF computation and signal generation.

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VI. CONCLUSIONS

A. SUMMARY OF FINDINGS

There were two goals for this thesis. One was to develop MATLAB software that implements CAF techniques to efficiently compute the TDOA and FDOA between two sampled signals. The second goal was to develop software capable of generating pairs of signals that are based upon user-defined parameters and emitter-collector geometries. The resulting programs, listed in Appendices A and B, achieved these two goals.

Many real-world collection systems utilize CAF techniques to locate radio frequency transmitters. These systems enjoy the benefit of enormous computing resources (e.g., supercomputers) with which to accomplish the CAF processing. The software developed for this thesis (Appendix A), however, provides a new capability for users to conduct CAF computations with the much more limited processing power of a desktop PC. In order to minimize processing burden, the CAF software splits the computations into coarse and fine modes. The coarse mode implements the algorithm described in [1]. An analysis of the computational complexities of three direct CAF calculation methods showed that utilizing an explicit summation is the most efficient method for the fine mode. Limitations of a desktop PC may restrict the size of the sampled signals that can be processed, but realistic simulations are definitely possible.

The software in Appendix B provides a new capability for users to produce realistic BPSK signal sets that might be transmitted and collected by real-world systems. This capability, combined with the CAF software, allows a user to model practical systems to determine whether or not particular signals could be detected and/or their emitters located. For example, a proposed LPD emitter could be simulated to analyze its effectiveness against CAF processing. These programs could be useful in many other applications as well.

B. FUTURE WORK

There are a number of ways that future research could build upon the work described in this thesis. One could analyze the theoretical processing gain provided by the CAF, and compare it to the results of actual test cases. For the CAF software, the coarse mode described in section III.B.1 only works for positive TDOAs. The coarse algorithm could be reviewed and possibly modified to handle negative TDOAs. During coarse mode processing in CAF.m, the estimations of TDOA and FDOA are associated with the single largest magnitude computed. In reality, coarse mode processing can produce multiple peaks and the correct TDOA and FDOA could be associated with any of them. The coarse algorithm in CAF.m could be modified so that it processes more than just the largest peak. This would require determining a threshold (based upon noise and other factors), above which a peak would be considered a candidate.

The fine mode in CAF.m could be automated so that the program itself would decide to what degree of resolution the TDOA and FDOA should be computed, rather than have the user decide how many times to run the fine mode. Reference [1] describes how the standard deviation of the CAF surface in the TDOA and FDOA directions can be used to determine the maximum degree of accuracy that can be expected. The standard deviations are dependent upon bandwidths, total collection time, and signal-to-noise ratios. Defining the maximum accuracy then sets the maximum resolution required for computations.

There are a couple of ways that the signal generation software could be enhanced. For example, it could be expanded to be able to generate other kinds of signals. Sig_gen.m is written so that it would be straightforward to add the ability to generate higher-order PSK signals (4-PSK, etc.). Other signal types might be added as well (Amplitude-Shift-Keying, etc.). Also, while the 3-D Cartesian coordinate system used in sig_gen.m provides a fairly realistic model, especially for short collect times, the program could be modified to use earth-centered coordinates instead. This would be complicated, but would clearly make simulations even more realistic.

APPENDIX A: MATLAB CODE – CAF SOFTWARE

A. "CAF.M"

```
function [TDOA, FDOA] = CAF(S1, S2, Max_f, fs, Max_t);

% ***************************************************************
% CAF takes as inputs two sampled signal vectors (S1 & S2) in analytic
%       signal format, the maximum expected FDOA in Hertz (Max_f), the
%       sampling frequency used to generate S1 & S2 (fs), and the maximum
%       expected TDOA in seconds (Max_t).  The function then utilizes
%       Stein's method in [1] to compute coarse estimations of TDOA and
%       FDOA between S1 & S2.  Finally, "fine mode" calcualtions are made
%       to compute the final TDOA and FDOA, which are returned to the
%       user via the output arguments.

% Written by:  LCDR Joe J. Johnson, USN
% Last modified:  17 September 2001
% ***************************************************************

clc;

N = length(S1);
S1 = reshape(S1,N,1);      % Ensures signals are column vectors due to
S2 = reshape(S2,N,1);      % Matlab's better efficiency on columns

S1_orig = S1;                 % Want to preserve original input signals
S2_orig = S2;                 % for later use; S1 & S2 will be
                              % manipulated in the fine mode below.


% The following while loop ensures that the sub-block size, N1, is
% large enough to ensure proper resolution.  If Max_f/fs*N1 were
% less than 1, then the Freq calculated at the end would always be
% + or - 1/N1!  2^19 = 524288 is about the limit for efficient
% processing speed.
N1=1024;
while (Max_f/fs*N1 < 2) & (N1 < 2^19)
  N1 = 2*N1;
end

N2=N1/2;
```

```
if N1 > N                         % For cases where resolution calls for
  S1 = [S1;zeros(N1-N,1)];        % a sub-block size larger than the
  S2 = [S2;zeros(N1-N,1)];        % signal vectors, pad the vectors with
  N = N1;                         % zeros so that they have a total of
end                               % N1 elements.

% Want magnitude of Max_f, since +&- will be used below
Max_f = abs(Max_f);
Number_of_Blocks = length(S1)/N1;          % Number of sub-blocks to break
                                           % the signal into

Min_v = floor(-Max_f/fs*N1);            % Smallest freq bin to search
Max_v = -Min_v;                         % Largest freq bin to search
v_values = Min_v : Max_v;               % Vector of all bins to search

Max_samples = Max_t * fs;         % Maximum number of samples to search

% Finds max number of block shifts (q) that must occur for each
% R and v below.
if Max_samples > N2
    q_max = min(ceil((Max_samples - N2)/N1),Number_of_Blocks-1);
else q_max = 0;
end

x=0;
divisors = Number_of_Blocks:-1:1;       % Used to scale "temp" below...


% ***************************************************************
% COARSE MODE computations.
% ***************************************************************

for v = 1:length(v_values)
  temp(1:N1,1:q_max+1)=0;             % Initializing -- saves time....
  for R = 0:Number_of_Blocks-1

    % temp1 is the FFT of the R'th block of S1, shifted by "v" bins.
    temp1 = fftshift(fft(S1(1+R*N1 : N1*(R+1))));
    temp1 = shiftud(temp1,v_values(v),0);
    for q = 0:q_max
      % R+q cannot exceed the number of sub-blocks
      if R + q > Number_of_Blocks-1 break
      end
```

```
        % FFT of the (R+q)'th block of S2
        temp2 = fftshift(fft([S2(1+(R+q)*N1 : N2 + N1*(R+q));...
                        zeros(N2,1)]));

        % Multiplies temp1 & temp2, FFTs the product, then adds to
        % previous values for the same value of q (but different R)
        temp(:,q+1) = temp(:,q+1) + ...
                             abs(fftshift(fft(temp1.*conj(temp2))));
      end
    end

    % Each value of q was used a different # of times, so they must be
    % scaled properly.
    for q_index = 1:q_max+1
      temp(:,q_index) = temp(:,q_index) / divisors(q_index);
    end

    % If combination of current v and any q provides a greater value
    % than the previous max, then remember m, Q, & V.
    if max(max(temp))>x
      x = max(max(temp));
      [m Q] = find(temp == max(max(temp)));

      % Must do this since q starts at 0, but Matlab doesn't allow for
      % zero indexing.
      Q = Q - 1;
      V = v_values(v);
    end
end

% Coarse estimate of TDOA (in # of samples)
TDOA_Coarse = Q * N1 + (-N2+1 + m);

% Coarse estimate of FDOA (in Freq Bin #)
FDOA_Coarse = V/N1*N;


% The following 3 lines can be used to display the coarse estimates,
% if desired.

%disp(['The coarse TDOA estimate is: ', num2str(TDOA_Coarse),...
%        ' samples.']);
%disp(['The coarse FDOA estimate is: ', num2str(FDOA_Coarse/N),...
%      ' (digital frequency).']);
% ******************************************************************
% FINE MODE computations.
% ******************************************************************

S2 = conj(S2);            % S2 is conjugated in basic CAF definition


% Vector of freq "bins" to use (DON'T have to be integers!!)
k_val = FDOA_Coarse-10 : FDOA_Coarse+10;

% Vectors of TDOAs to use (must be integers)
tau_val = TDOA_Coarse-10 : TDOA_Coarse+10;

done = 0;
multiple = 1;
decimal = 0;
while ~done          % Fine mode iterations continue until user is done.

  % Initialize to make later computations faster
  amb(length(k_val),length(tau_val))=0;
  Ntemp = N * multiple;
  for k = 1:length(k_val)        % Must loop through all values of k

    % Vector of complex exponentials that will be used
    exponents = exp(-j*2*pi*k_val(k)/Ntemp*(0:Ntemp-1)');

    % Must loop through all potential TDOAs
    for t = 1:length(tau_val)

      % S2 is shifted "tau" samples
      S2temp = shiftud(S2,tau_val(t),0);

      % Definition of CAF summation
      temp = abs(sum(S1.*S2temp.*exponents));

      % Save CAF magnitude for the values of k & t
        amb(k,t)=temp;
    end
  end

  [k, t]=find(amb==max(max(amb)));      % Find the peak of the CAF matrix


  TDOA = tau_val(t); % TDOA and FDOA associated with the peak of the
  FDOA = k_val(k);         % CAF plane.  These represent the final TDOA
                           % & FDOA estimates.
  % The results are displayed.
  disp(' ');disp(' ');disp(' ');
  disp(['The TDOA is ', num2str(TDOA/multiple), ' samples']);
  disp(['        or ', num2str(TDOA/(multiple*fs)), ' seconds.']);
  disp(' ');
  disp(['The resolution is ', num2str(0.5/...
                                          (multiple*fs)),' seconds.']);
  disp(' ');disp(' ');

  disp(['The FDOA is ', num2str(FDOA/N),...
                                      ' in digital frequency (k/N)']);
  disp(['        or ', num2str(FDOA/N*fs), ' Hz.']); disp(' ');
  disp(['The resolution is ', num2str(0.5*...
                                          (10^decimal)/N*fs), ' Hz.']);
  disp(' ');disp(' ');disp(' ');


  % If the signal length exceeds 524288 elements, max processing
  % capability has been achieved, and the user will not be given
  % the option of refining TDOA any further.
  if Ntemp >= 2^19
    disp('Maximum TDOA processing capability has been achieved.')
    doneT = 1;
  else doneT = 0;
  end

  % User chooses whether to compute more accurate TDOA &/or
  % FDOA, or to stop fine mode computations.
  disp('Do you desire a solution with finer resolution?');
  disp('Select one of the following:'); disp(' ');

  if ~doneT
    disp('1.  Finer resolution for TDOA.');
  else disp(' ');
  end

  disp('2.  Finer resolution for FDOA.');

  if ~doneT
    disp('3.  Finer resolution for both TDOA and FDOA.');
  else disp(' ');
  end

  disp('4.  The TDOA and FDOA resolutions are fine enough.');
  disp(' ');
```

```
    choice = input('What is your selection?  ');

    switch choice

    % TDOA is refined by resampling the signals at twice the
    % previous sampling rate.  Increases resolution two-fold.
    case 1
      if ~doneT
        multiple = multiple*2;
        S1 = interp(S1, 2);
        S2 = interp(S2, 2);
        tau_val = TDOA*2 - 1 : TDOA*2 + 1;
      else done = 1;
      end
      clc;

    % FDOA resolution is improved by a factor of 10.
    case 2
      decimal = decimal - 1;
      k_val = FDOA - 5*10^decimal : 10^decimal : FDOA + 5*10^decimal;
      clc;

    % Both TDOA and FDOA resolutions are improved.
    case 3
      if ~doneT
        multiple = multiple*2;
        S1 = interp(S1, 2);
        S2 = interp(S2, 2);
        tau_val = TDOA*2 - 1 : TDOA*2 + 1;

        decimal = decimal - 1;
        k_val = FDOA - 5*10^decimal : 10^decimal : FDOA + ...
                                                          5*10^decimal;
      else done = 1;
      end
      clc;
    otherwise
      done = 1;
    end

    if done
      disp(' ');disp(' '); disp('TDOA & FDOA estimation complete.');
    end
  end
```

```
% If user wants to see the CAF surface graphically, a call to
% CAF_peak is made.
disp(' ');%disp(' ');disp(' ');
choice = input...
  ('Would you like to see the CAF peak graphically (Y or N)?  ','s');
choice = upper(choice);

switch choice
case 'Y'
  caf_peak(S1_orig, S2_orig, floor(TDOA/multiple) - 50, ...
    floor(TDOA/multiple) + 50, (FDOA-20)/N, (FDOA+20)/N, fs);
end


TDOA = TDOA/(multiple*fs);              % Returns TDOA in seconds.
FDOA = FDOA/N*fs;                           % Returns FDOA in Hertz.
disp('Program Complete.');
```

B. “CAF_PEAK.M”

```
function [TDOA, FDOA, MaxAmb, Amb] = ...
                  CAF_peak(S1, S2, Tau_Lo, Tau_Hi, Freq_Lo, Freq_Hi, fs);

% ******************************************************************
% CAF_peak(S1, S2, Tau_Lo, Tau_Hi, Freq_Lo, Freq_Hi) takes as input:
%    two signals (S1, S2) that are row or column vectors; a range of
%    time delays (in samples) to search (Tau_Lo, Tau_Hi must be
%    integers between -N & +N); a range of digital frequencies (in
%    fractions of sampling frequency) to search (Freq_Lo, Freq_Hi must
%    be between -1/2 and 1/2, or -(N/2)/N and (N/2)/N, where N is the
%    length of the longer of the two signal vectors); and the sampling
%    frequency, fs.
%
%    The function computes the Cross Ambiguity Function of the two
%    signals. Four plots are produced which represent four different
%    views of the Cross Ambiguity Function magnitude versus the input
%    Tau and Frequency Offset ranges.
%
%    The function returns the scalars TDOA, FDOA, and MaxAmb, where
%    TDOA & FDOA are the values of Time Delay and Frequency Offset
%    that cause the Cross Ambiguity Function to peak at a magnitude
%    of MaxAmb.  Amb is the matrix of values representing the CAF
%    surface.
% Written by:  LCDR Joe J. Johnson, USN
% Last modified:  26 August 2001
% ************************************************************************


% Ensures that the user enters all SIX required arguments.
if (nargin < 6)
   error...
   ('6 arguments required: S1, S2, Tau_Lo, Tau_Hi, Freq_Lo, Freq_Hi');
end

% Ensures that both S1 & S2 are row- or column-wise vectors.
if ((size(S1,1)~=1)&(size(S1,2)~=1)) | ((size(S2,1)~=1)&...
                                              (size(S2,2)~=1))
   error('S1 and S2 must be row or column vectors.');
end

N1 = length(S1);
N2 = length(S2);
S1 = reshape(S1,N1,1);         % S1 & S2 are reshaped into column-wise
S2 = reshape(S2,N2,1);         % vectors since MATLAB is more efficient
                               % when manipulating columns.

S1 = [S1;zeros(N2-N1,1)];      % Ensure that S1 & S2 are the same size,
S2 = [S2;zeros(N1-N2,1)];      % padding the smaller one w/ 0s as neeeded.


% This WHILE loop simply ensures that the length of S1 & S2 is a power
% of two.  If not, the vectors are padded with 0s until their length
% is a power of two.  This is not required, but it takes advantage of
% the fact that MATLAB's FFT computation is significantly faster for
% lengths which are powers of two!
while log(length(S1))/log(2) ~= round(log(length(S1))/log(2))
   S1(length(S1)+1) = 0;
   S2(length(S2)+1) = 0;
end

N = length(S1);

% Ensures that the Tau values entered are in the valid range.
if abs(Tau_Lo)>N | abs(Tau_Hi)>N
   error('Tau_Lo and Tau_Hi must be in the range -N to +N.');
end
```

```
% Ensures that Tau values entered by the user are integers.
if (Tau_Lo ~= round(Tau_Lo)) | (Tau_Hi ~= round(Tau_Hi))
  error('Tau_Lo and Tau_Hi must be integers.')
end

% Ensures that the Frequency values entered are in the valid range.
if abs(Freq_Lo)>1/2 | abs(Freq_Hi)>1/2
  error('Freq_Lo and Freq_Hi must be in the range -.5 to +.5');
end

% Ensures that the lower bounds are less than the upper bounds.
if (Tau_Lo > Tau_Hi) | (Freq_Lo > Freq_Hi)
  error('Lower bounds must be less than upper bounds.')
end

% Freq values converted into integers for processing.
Freq_Lo = round(Freq_Lo*N);
Freq_Hi = round(Freq_Hi*N);

% Creates vectors for the Tau & Freq values entered by the user. Used
% for plotting...
TauValues = [Tau_Lo:Tau_Hi];
FreqValues = [Freq_Lo:Freq_Hi]/N;

% The IF statement calculates the indices required to isolate the
% user-defined frequencies from the FFT calculations below.
if Freq_Lo < 0 & Freq_Hi < 0
  Neg_Freq = (N+Freq_Lo+1:N+Freq_Hi+1);
  Pos_Freq = [];
elseif Freq_Lo < 0 & Freq_Hi >= 0
  Neg_Freq = (N+Freq_Lo+1:N);
  Pos_Freq = (1:Freq_Hi+1);
else
  Neg_Freq = [];
  Pos_Freq = (Freq_Lo+1:Freq_Hi+1);
end


% This FOR loop actually calculates the Cross Ambiguity Function for
% the given range of Taus and Frequencies.  Note that an FFT is
% performed for each Tau value and then the frequencies of interest
% are isolated using the Neg_Freq and Pos_Freq vectors obtained above.
% For each value of Tau, the vector S2 is shifted Tau samples using a
% call to the separate function "SHIFTUD".  Samples shifted out are
% deleted and zeros fill in on the opposite end.
% Initializing Amb with 0s makes computations much faster.
Amb=zeros(length(Neg_Freq)+length(Pos_Freq),length(TauValues));
for t = 1:length(TauValues)
  temp = fft((S1).*conj(shiftud(S2,TauValues(t),0)));
  Amb(:,t) = [temp(Neg_Freq);temp(Pos_Freq)];
end

% Only interested in the Magnitude of the Cross Ambiguity Function.
Amb = abs(Amb);


% The following will remove any spike that occurs at Tau = FreqOff = 0.
% This may be desired in some cases, especially when the spike at (0,0)
% is due to correlation of the two signals' noise components.  The
% spike, of course, could also indicate that the two signals have no
% TDOA or FDOA between them.

% if find(TauValues == 0) & find(FreqValues == 0)
%   Amb(find(FreqValues==0),find(TauValues==0)) = 0;
% end

%clc;                   %Clears the MATLAB command window.
% The four different views of the Cross Ambiguity Function plots are
% created here.

figure        % This one is the 3-D view
mesh(TauValues/fs,FreqValues*fs,Amb);
    xlabel('TDOA (Seconds)');ylabel('FDOA (Hertz)');
    zlabel('Magnitude');
    title('Cross Ambiguity Function');

figure
subplot(2,1,1)      % This one is the 2-D view along the TDOA axis
mesh(TauValues/fs,FreqValues*fs,Amb);
    xlabel('TDOA (Seconds)');
    zlabel('Magnitude');
    view(0,0);

subplot(2,1,2)      % This one is the 2-D view along the FDOA axis
mesh(TauValues/fs,FreqValues*fs,Amb);
    ylabel('FDOA (Hertz)');
    zlabel('Magnitude');
    view(90,0);
```

```
%This one is a 2-D view looking down on the plane
figure
mesh(TauValues/fs,FreqValues*fs,Amb);
    xlabel('TDOA (Seconds)');ylabel('FDOA (Hertz)');
    zlabel('Magnitude');
    title('Cross Ambiguity Function');
    view(0,90);


% Finds the indices of the peak value.
[DFO, DTO] = find(Amb==max(max(Amb)));

TDOA = TauValues(DTO);        % Finds the actual value of the TDOA.
FDOA = FreqValues(DFO);       % Finds the actual value of the FDOA.
MaxAmb = max(max(Amb));       % Finds the actual Magnitude of the peak.


% The remaining lines will display the numerical results of the
% TDOA & FDOA, if desired.  Since the FFT method was used for the
% calculations, the TDOA is accurate only to within +/- 0.5 samples,
% and the FDOA is accurate to within +/- 0.5/N in digital frequency.

% disp(' '); disp(' ');
% disp(['The TIME LAG (TDOA) is:  ',num2str(TDOA),' Samples.']);
% disp(' ');
% disp(['The FREQ OFFSET (FDOA) is:  ',num2str(FDOA),...
%     ' (Fraction of Fs).']);
% disp(' '); disp(['Maximum Magnitude = ',num2str(MaxAmb)]);
% disp(' ');disp('-----------------------------');
% disp('NOTE:  If the CAF plot has secondary peaks whose magnitudes');
% disp('     are within about 80% of the Main Peak''s magnitude,');
% disp('     then the above results may be unreliable.  Likely');
% disp('     reasons: The true peak is not within the range of,');
% disp('          Taus & Freq Offsets that you entered or the signals');
% disp('     may be too noisy to detect the peak.');
```

C. "SHIFTUD.M"

```
function y=shiftud(a,n,cs)

% *****************************************************************
% SHIFTUD Shift or Circularly Shift Matrix Rows
% SHIFTUD(A,N) with N<0 shifts the rows of A DOWN N rows.
% The first N rows are replaced by zeros and the last N
% rows of A are deleted.
%
% SHIFTUD(A,N) with N>0 shifts the rows of A UP N rows.
% The last N rows are replaced by zeros and the first N
% rows of A are deleted.
%
% SHIFTUD(A,N,C) where C is nonzero performs a circular
% shift of N rows, where rows circle back to the other
% side of the matrix.  No rows are replaced by zeros.
%
% Copyright (c) 1996 by Prentice-Hall, Inc. – Reference [9]
% ****************************************************************

if nargin<3, cs=0; end      % If no third argument, default is False
cs=cs(1);                   % Make sure third argument is a scalar
[r,c]=size(a);              % Get dimensions of input
dn=(n<=0);                  % dn is True if shift is down
n=min(abs(n),r);            % Limit shift to less than rows

if n==0|(cs&n==r)                   % Simple no shift case
  y=a;
elseif ~cs&dn                       % No circular and down
  y=[zeros(n,c); a(1:r-n,:)];
elseif ~cs&~dn                      % No circular and up
  y=[a(n+1:r,:); zeros(n,c)];
elseif cs&dn                        % Circular and down
  y=[a(r-n+1:r,:); a(1:r-n,:)];
elseif cs&~dn                       % Circular and up
  y=[a(n+1:r,:); a(1:n,:)];
end
```

APPENDIX B: MATLAB CODE – SIGNAL GENERATION SOFTWARE

A. "SIG_GEN.M"

```
function [Sa1, Sa2, S1, S2] = sig_gen;

% ********************************************************************
% SIG_GEN generates BPSK signal pairs based upon user-defined param-
%             eters and Cartesian emitter-collector geometries.  There are
%             no input arguments, since the function queries the user for
%             all required inputs.  The function returns four vectors:
%             Sa1, Sa2, S1 & S2.  These are the Analytic Signal represen-
%             tations of the two generated signals, and the Real represen-
%             tations of the two signals, respectively.
%
% Written by:  LCDR Joe J. Johnson, USN
% Last modified:  26 August 2001

% ********************************************************************

clc;
disp(' ');
disp('All positions and velocites must be entered in vector format,');
disp('e.g., [X Y Z] or [X, Y, Z] (including the brackets).');
disp(' ');

Pc1(1,:) = input...
        ('Collector 1''s POSITION Vector at time 0 (in meters)?  ');
Vc1 = input('Collector 1''s VELOCITY Vector (in m/s)?  '); disp(' ');

Pc2(1,:) = input...
        ('Collector 2''s POSITION Vector at time 0 (in meters)?  ');
Vc2 = input('Collector 2''s VELOCITY Vector (in m/s)?  '); disp(' ');

Pe(1,:) = input...
            ('Emitter''s POSITION Vector at time 0 (in meters)?  ');
Ve = input('Emitter''s VELOCITY Vector (in m/s)?  '); disp(' ');

% f0 and fs are the same for BOTH collectors!
f0 = input('Carrier Frequency (in Hz)?  ');
fs = input('Sampling Frequency (in Hz)?  ');
Ts = 1/fs;                            % Calculates Sample Period

Rsym = input('Symbol Rate (in symbols/s)?  '); disp(' ');
Tsym = 1/Rsym;                        % Calculates Symbol Period
```

```
N = input('How many samples?  '); disp(' ');

Es_No1 = input('Desired Es/No at Collector 1 (in dB)?  ');
Es_No1 = 10^(Es_No1/10);            % Converts from dB to ratio

Es_No2 = input('Desired Es/No at Collector 2 (in dB)?  ');
disp(' ');
Es_No2 = 10^(Es_No2/10);            % Converts from dB to ratio


Pc1 = [Pc1; zeros(N-1, 3)];         % Initializing all the matrices makes
Pe1 = zeros(N, 3);                  % later computations much faster.
Pc2 = [Pc2; zeros(N-1, 3)];
Pe2 = zeros(N, 3);
t1 = zeros(1, N);
t2 = zeros(1, N);
S1 = zeros(1, N);
S2 = zeros(1, N);

A = 1;                     % Amplitude of Signal
c = 2.997925e8;            % Speed of light in m/s
Ps = (A^2)/2;              % Power of Signal

sigma1 = sqrt(Ps*Tsym/Es_No1);      % Calculate Noise Amplification fac-
sigma2 = sqrt(Ps*Tsym/Es_No2);      % tors using Es/No = Ps*Tsym/sigma^2

Noise1 = sigma1.*randn(N, 1);       % Random Noise values for Signal 1
Noise2 = sigma2.*randn(N, 1);       % Random Noise values for Signal 2


% Builds the position vectors for the two collectors
for index = 2 : N
  Pc1(index,:) = Pc1(index - 1,:) + Ts*Vc1;
  Pc2(index,:) = Pc2(index - 1,:) + Ts*Vc2;
end


% While loop determines first elements of Pe1 and t1.  t1(1) is the
% time AT THE EMITTER that produces the 1st sample received at
% collector 1!  Pe1(1,:) is the position of the emitter when it
% produces the 1st sample received by collector 1.

temp = inf;          % Ensures while loop executes at least once
t1(1) = 0;
tempPe = Pe(1,:);
```

```
while abs(temp - t1(1)) > 1/f0
  temp = t1(1);
  t1(1) = -norm(Pc1(1,:) - tempPe) / c;
  tempPe = Pe(1,:) + t1(1)*Ve;
end
Pe1(1,:) = tempPe;


% While loop determines first elements of Pe2 and t2.  t2(1) is the
% time AT THE EMITTER that produces the 1st sample received at
% collector 2!  Pe2(1,:) is the position of the emitter when it
% produces the 1st sample received by collector 2.

temp = inf;           % Ensures while loop executes at least once
t2(1) = 0;
tempPe = Pe(1,:);
while abs(temp - t2(1)) > 1/f0
  temp = t2(1);
  t2(1) = -norm(Pc2(1,:) - tempPe) / c;
  tempPe = Pe(1,:) + t2(1)*Ve;
end
Pe2(1,:) = tempPe;


% Determines the earliest time at the emitter for this pair of signals.
StartPoint = min(t1(1), t2(1));


% Next 2 lines determine offsets needed for signals 1 & 2 to enter the
% phase vector (P).  This simply ensures proper line up so that bit
% changes occur at the right times.
SymbolIndex1 = 1 + floor(abs(t1(1) - t2(1))/Tsym) * (t1(1)>t2(1));
SymbolIndex2 = 1 + floor(abs(t1(1) - t2(1))/Tsym) * (t2(1)>t1(1));


for index = 2 : N                    % Builds the Pe1 and t1 vectors
  temp = inf;
  t1(index) = 0;

  % 1st guess is that emitter will advance exactly Ts seconds.
  tempPe = Pe1(1,:) + (t1(index -1) + Ts)*Ve;
  % While loop iteratively determines actual time & position for
  % emitter, based on instantaneous geometry.
  while abs(temp - t1(index)) > 1/f0
    temp = t1(index);
    t1(index) = (index - 1)*Ts - norm(Pc1(index,:) - tempPe) / c;

    % Due to negative times, must multiply Ve by ELAPSED time!
    tempPe = Pe1(1,:) + abs(t1(1)-t1(index))*Ve;
  end
  Pe1(index,:) = tempPe;
end


for index = 2 : N                    % Builds the Pe2 and t2 vectors
  temp = inf;
  t2(index) = 0;

  % 1st guess is that emitter will advance exactly Ts seconds.
  tempPe = Pe2(1,:) + (t2(index -1) + Ts)*Ve;

  % While loop iteratively determines actual time & position for
  % emitter, based on instantaneous geometry.
  while abs(temp - t2(index)) > 1/f0
    temp = t2(index);
    t2(index) = (index - 1)*Ts - norm(Pc2(index,:) - tempPe) / c;

    % Due to negative times, must multiply Ve by ELAPSED time!
    tempPe = Pe2(1,:) + abs(t2(1)-t2(index))*Ve;
  end
  Pe2(index,:) = tempPe;
end



% Could change this seed to whatever you want; or could have user
% define it as an input.  This just ensures, for simulation purposes
% that every time the program is run, the BPSK signals created will
% have the same random set of data bits.
rand('seed',5);

% Create enough random #'s to figure phase shift (data bits)
r = rand(N,1);
P = (r > 0.5)*0 + (r <= 0.5)*1;     % Since BPSK, random # determines
                                     % if phase is 0 or pi
% Building Xmitted Signal #1 vector...  These represent the pieces of
% the signal that were transmitted by the emitter to arrive at
% Collector 1 at its sample intervals.

S1(1) = A*cos(2*pi*f0*t1(1) + P(SymbolIndex1)*pi) + Noise1(1);

% The if statement inside the loop changes the data bit if the time
% has advanced into the next symbol period.
for index = 2 : N
  if t1(index) - StartPoint >= (SymbolIndex1) * Tsym
    SymbolIndex1 = SymbolIndex1 + 1;
  end
  S1(index) = A*cos(2*pi*f0*t1(index) + P(SymbolIndex1)*pi) + ...
              Noise1(index);
end

Sa1 = hilbert(S1); % Calculates the ANALYTIC SIGNAL of S1.  To
                   % compute the COMPLEX ENVELOPE, multiply Sa1
                   % by .*exp(-j*2*pi*f0.*t1);



% Building Xmitted Signal #2 vector...  These represent the pieces of
% the signal that were transmitted by the emitter to arrive at
% Collector 2 at its sample intervals.

S2(1) = A*cos(2*pi*f0*t2(1) + P(SymbolIndex2)*pi) + Noise2(1);

% The if statement inside the loop changes the data bit if the time
% has advanced into the next symbol period.
for index = 2 : N
  if t2(index) - StartPoint >= (SymbolIndex2) * Tsym
    SymbolIndex2 = SymbolIndex2 + 1;
  end
  S2(index) = A*cos(2*pi*f0*t2(index) + P(SymbolIndex2)*pi) + ...
              Noise2(index);
end

Sa2 = hilbert(S2); % Calculates the ANALYTIC SIGNAL of S2.  To
                   % compute the COMPLEX ENVELOPE, multiply Sa2
                   % by .*exp(-j*2*pi*f0.*t2);
% This function call simply calculates and displays the expected TDOAs
% and FDOAs at the Beginning and End of the collection time.

tdoa_fdoa(f0,Pe1(1,:),Pe1(N,:),Pe2(1,:),Pe2(N,:),Ve,Pc1(1,:),...
                    Pc1(N,:),Vc1,Pc2(1,:),Pc2(N,:),Vc2);
```

B. "TDOA_FDOA.M"

```
function [TDOA_b, TDOA_e, FDOA_b, FDOA_e] = tdoa_fdoa(f0,Pe1_b,...
                    Pe1_e,Pe2_b,Pe2_e,Ve, Pc1_b,Pc1_e,Vc1,Pc2_b,Pc2_e,Vc2)

% ****************************************************************
% TDOA_FDOA is for use with SIG_GEN.m in helping to determine what the
%                     expected TDOA and FDOA are for two signal vectors.
%
% The function takes the following input arguments:
%
%             f0 -- carrier frequency (assumed to be equal for both signals)
%        Pe1_b -- [X Y Z] Emitter position WRT Collector 1 at 1st sample
%        Pe1_e -- [X Y Z] Emitter position WRT Collector 1 at last sample
%        Pe2_b -- [X Y Z] Emitter position WRT Collector 2 at 1st sample
%        Pe2_e -- [X Y Z] Emitter position WRT Collector 2 at last sample
%           Ve -- [X Y Z] Emitter velocity
%        Pc1_b -- [X Y Z] Collector 1 position at 1st sample
%        Pc1_e -- [X Y Z] Collector 1 position at last sample
%          Vc1 -- [X Y Z] Collector 1 velocity
%        Pc2_b -- [X Y Z] Collector 2 position at 1st sample
%        Pc2_e -- [X Y Z] Collector 2 postion at last sample
%          Vc2 -- [X Y Z] Collector 2 velocity
%
% The output variables are the TDOA at the beginning, TDOA at the
% end, FDOA at the beginning, and FDOA at the end, respectively.

% Written by:  LCDR Joe J. Johnson, USN
% Last modified:  26 August 2001
% ****************************************************************

c = 2.997925e8;              % Speed of light

% The next two lines calculate the Doppler shifts between the emitter
% and Collector 1 & Collector 2, respectively, at the BEGINNING of the
% collection (i.e., at the instant of the first sample).

doppler1_b = f0/c * dot(Ve-Vc1, Pe1_b-Pc1_b) / norm(Pe1_b - Pc1_b);
doppler2_b = f0/c * dot(Ve-Vc2, Pe2_b-Pc2_b) / norm(Pe2_b - Pc2_b);
```

```
% Calculates the FDOA at the BEGINNING of collection time.

FDOA_b = doppler1_b - doppler2_b;


% Calculates Doppler shifts between emitter and each collector at the
% END of the collection time (i.e., at instant of the last sample).
doppler1_e = f0/c * dot(Ve-Vc1, Pe1_e-Pc1_e) / norm(Pe1_e - Pc1_e);
doppler2_e = f0/c * dot(Ve-Vc2, Pe2_e-Pc2_e) / norm(Pe2_e - Pc2_e);

% Calculates the FDOA at the END of collection time
FDOA_e = doppler1_e - doppler2_e;


% Calculates the TDOA between the two collectors at the BEGINNING
% and END of collection time.

TDOA_b = (norm(Pe2_b - Pc2_b) - norm(Pe1_b - Pc1_b)) / c;
TDOA_e = (norm(Pe2_e - Pc2_e) - norm(Pe1_e - Pc1_e)) / c;


% Displays the results in the command window.

disp(' ');disp(' ');disp(' ');
disp(['At the START of the Collection, TDOA = ',num2str(TDOA_b),...
    ' seconds.']);
disp(['                               FDOA = ',num2str(FDOA_b),...
    ' Hertz.']);

disp(' ');disp(' ');
disp(['At the END of the Collection, TDOA = ',num2str(TDOA_e),...
    ' seconds.']);
disp(['                             FDOA = ',num2str(FDOA_e),...
    ' Hertz.']);
```

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